AD-767 475

SPECIALISTS MEETING ON HELICOPTER ROTOR LOADS PREDICTION METHODS

Advisory Group for Aerospace Research and Development Paris, France

August 1973

**DISTRIBUTED BY:** 



National Technical Information Service
U. S. DEPARTMENT OF COMMERCE
5285 Port Royal Road, Springfield Va. 22151

GARD-CP-122

AD 767475

AGARD-CP-122

ADVISORY GROUP FOR AEROSPACE RESEARCH & DEVELOPMENT

7 RUE ANCELLE: 92200 NEUILLY SUR SEINE FRANCE

AGARD CONFERENCE PROCEEDINGS No. 122

Specialists Meeting on Helicopter Rotor Loads Prediction Methods

DISTRIBUTION STATEMENT A

Approved for public release;

NORTH ATLANTIC TREATY ORGANIZATION



DISTRIBUTION AND AVAILABILITY ON BACK COVER

Reproduced by NATIONAL TECHNICAL

# NORTH ATLANTIC TREATY ORGANIZATION ADVISORY GROUP FOR AEROSPACE RESEARCH AND DEVELOPMENT (ORGANISATION DU TRAITE DE L'ATLANTIQUE NORD)

AGARD Conference Proceedings No.122

# SPECIALISTS MEETING ON HELICOPTER ROTOR LOADS PREDICTION METHODS



#### THE MISSION OF AGARD

The mission of AGARD is to bring together the leading personalities of the NATO nations in the fields of science and technology relating to aerospace for the following purposes:

- Exchanging of scientific and technical information;
- Continuously stimulating advances in the aerospace sciences relevant to strengthening the common defence posture:
- Improving the co-operation among member nations in aerospace research and development;
- Providing scientific and technical advice and assistance to the North Atlantic Military Committee in the field of aerospace research and development;
- Rendering scientific and technical assistance, as requested, to other NATO bodies and to member nations
  in connection with research and development problems in the aerospace field;
- Providing assistance to member nations for the purpose of increasing their scientific and technical potential;
- Recommending effective ways for the member nations to use their research and development capabilities for the common benefit of the NATO community.

The highest authority within AGARD is the National Delegates Board consisting of officially appointed senior representatives from each member nation. The mission of AGARD is carried out through the Panels which are composed of experts appointed by the National Delegates, the Consultant and Exchange Program and the Aerospace Applications Studies Program. The results of AGARD work are reported to the member nations and the NATO Authorities through the AGARD series of publications of which this is one.

Participation in AGARD activities is by invitation only and is normally limited to citizens of the NATO nations.

The material in this publication has been reproduced directly from copy supplied by AGARD or the author.

Published Avaust 1973

533.662.6



Printed by Technical Editing and Reproduction Ltd Harford House, 7–9 Charlotte St, London. WIP 1HD

#### **PREFACE**

In the Spring of 1971 the Structures and Materials Panel of the Advisory Group for Aerospace Research and Development established a Working Group on Helicopter V/STOL Structures and Dynamics. The primary function of the Working Group was to develop means of enhancing the operational capability of helicopters and V/STOL aircraft, and to provide for better performance and reliability and reducing maintenance.

As an appraoch to the problem, the Working Group felt that a contribution to improve the prediction methods of dynamic loads on helicopters was urgently needed. In order to thoroughly survey the state of the art of loads prediction and to define areas for further research and development, the Working Group decided to organize a Specialists Meeting on "Helicopter Rotor Loads Prediction Methods". The meeting was subsequently held in conjunction with the 36th Meeting of the Structures and Materials Panel in Milan on 30-31 March 1973. The Conference Proceedings document for this Specialists Meeting contains a delineation of the analytical methods of predicting loads on the rotors and the airframe of helicopters, as presently used by eight different airframe manufacturers in the NATO Nations. These methods were critically compared by eight reviewers with special regard to the correlation of the analytical results with experimental experience. A general review of the state of the art was presented by Dr Loewy, who also pointed out those problems which still remain unsolved.

It is to be hoped that the results of the Specialists Meeting will stimulate efforts for further improvement of the analytical prediction methods for loads on helicopter rotors and airframes and will therefore contribute to an increase in the reliability of future generations of helicopters.

The Working Group is indebted to the authors and the reviewers, who by their valuable presentations contributed to the success of the meeting. Profound thanks are extended to Mr R.S.Berrisford and Mr F.Liard, Coordinators of the Working Group, and to Mr P.K.Bamberg, Executive of the Structures and Materials Panel, for their excellent work in the organization of the Conference and the preparation of the Proceedings of the Conference which were edited in the relatively short period of three months after the Specialists Meeting.

W.F.THIELEMANN Chairman of the Working Group on Helicopter V/STOL Structures and Dynamics

#### **CONFERENCE CHAIRMAN**

Professor W.THIELEMANN
D.F.V.L.R.
Institut für Flugzeugbau und Leichtbau
Braunschweig
Germany

#### **COORDINATORS**

R.S.BERRISFORD
Chief, Structures Division
U.S. Army Air Mobility Research
and Development Laboratory

Fort Eustis, Virginia U.S.A. F.LIARD
Chef de Département
Société Nationale Industrielle
Aérospatiale — Division Hélicoptères
Usine de Marignane, Marseille
France

#### STRUCTURES AND MATERIALS PANEL

**CHAIRMAN** 

Dr Th.GAYMANN Industrieanlagen-Betriebsgesellschaft Ottobrunn bei München Germany **DEPUTY CHAIRMAN** 

Professor F.NIORDSON
Technical University of Denmark
Department of Solid Mechanics
Lyngby, Denmark

#### PANEL EXECUTIVE

Dipl. Ing. Peter K.BAMBERG AGARD

#### HOST COORDINATOR

Colonel PALMIERI G. AGUSTA S.p.A. Cascina Costa 21013 Gallarate Italy

### CONTENTS

	Page
PREFACE	iii
CONFERENCE OFFICIALS	iv
	Reference
ROTARY WING DESIGN TECHNOLOGY by A.Z.Lemnios	1
CURRENT LOADS TECHNOLOGY FOR HELICOPTER ROTORS by R.Gabel	2
Review 1 by A.J.Sobey Review 2 by R.Piziali	R 1 R 2
DISCUSSIONS	D 1
CALCUL DES CHARGES SUR ROTOR D'HELICOPTERE par M.Gallot	3
HELICOPTER ROTOR LOADS PREDICTION by P.J.Arcidiacono and R.G.Carlson	4
Review 3 by J.L.Shipley	R 3
DISCUSSIONS	D II
ROTOR SYSTEM DESIGN AND EVALUATION USING A GENERAL PURPOSE HELICOPTER FLIGHT SIMULATION PROGRAM by R.L.Bennett	5
THE PREDICTION OF LOADING ACTIONS ON HIGH SPEED SEMI-RIGID ROTOR HELICOPTERS by K.T.McKenzie and D.A.S.Howell	6
by K.1.McKenzie and D.A.S.Howen	Ū
Review 4 by H.R.Velkoff Review 5 by R.Hirsch	R 4 R 5
DISCUSSIONS	D III
LOAD PREDICTION METHODS FOR HINGELESS ROTOR HELICOPTERS by G. Reichert	7
INTEGRATED ROTOR/BODY LOADS PREDICTION by R.M.Carlson and A.W.Kerr	8
Review 6 by K.H.Hohenemser Review 7 by J.L.Jenkins	R 6 R 7
DISCUSSIONS	D IV
Summary Analysis by R.G.Loewy	S

#### ROTARY WING DESIGN METHODOLOGY

hν

Andrew Z. Lemnios, Ph.D. Chief Research Engineer Kaman Aerospace Corporation Old Windsor Road Bloomfield, Connecticut 06002 USA

#### SUMMARY

A nonlinear aeroelastic blade loads analysis is described for calculating the coupled responses, airloads distributions, and performance of helicopter rotors. The analysis is divided into two major parts: (1) calculation of blade transient stability behavior by means of linearized, coupled equations of motion; (2) calculation of periodic blade dynamics and airloads distributions using fully coupled, nonlinear equations of motion. The analysis includes six response modes and two input control modes. The equations of motion include all nonlinear inertial coupling effects and nonlinear aero-dynamic effects such as reverse flow, Mach number variations, large induced flow angles, unsteady aerodynamics, and variable inflow. Additional features to the analysis are the inclusion of feedback mechanical coupling among the assumed modes and the inclusion of springs and dampers for each mode. Springs are also included for the two control systems in order that accurate control loads can be calculated. Unique to the analysis is a trim/optimization analysis that automatically changes control inputs until preselected trimmed flight conditions are obtained. The trim analysis applies to rotors with single controls or dual controls. The aeroelastic analysis was used to evaluate, design, and develop the new 101 Rotor for the HH2 helicopter. Significant improvements in stall speed, maximum speed, rotor power, and blade life have been demonstrated on the final design. The aeroelastic analysis was also used to evaluate a new rotor system with torsionally elastic biade and dual controls - the Controllable Twist Rotor. Comparisons are made between a CTR and a conventional rotor that are both sized to pret an assumed mission. The comparisons indicate significant advantages of the CTR in performance and blade dynamics.

#### INTRODUCTION

As each new generation of helicopters becomes more sophisticated, increased emphasis is put on improving the aerodynamic and structural efficiency of rotors. The accurate prediction of rotor blade loads and motions throughout the flight spectrum becomes increasingly important. An improvement in rotor efficiency comes about only by understanding the aeroelastic coupling effects within a rotor system. Detailed aeroelastic rotor analyses are required to achieve this understanding. In order to improve prediction techniques, the analyst must expand the mathematical model that represents the helicopter rotor by introducing additional degrees of freedom. An increase of the permissible modes of motion in the model introduces many nonlinear terms in the equations of motion. Solutions to these complete equations can be approximated by analytic or numerical methods.

In order to obtain analytical solutions to the governing equations of motion, past methods for solving a multi-degree-of-freedom rotor assumed small angular motions and linearizations of the resultant terms. Separate numerical methods were also developed which could handle large angular amplitudes and nonlinear terms for single-degree-of-freedom flapping motion. A brief state-of-the-art review of helicopter rotor blade airload and motion prediction techniques is presented in the following section.

The advent and widespread use of large, high-speed digital computers have introduced a tool with the capacity for obtaining numerical solutions to nonlinear equations of motion for systems with many degrees of freedom. Thus, the need has been eliminated for limiting the methods of solution to linearized multi-degree-of-freedom models or to nonlinear single-degree-of-freedom models.

The derivation of the complete nonlinear equations of motion for a multi-degree-of-freedom rotor requires extensive effort by the analyst and is susceptible to errors. In order to minimize the chance for errors during the derivation process, a technique was selected for deriving the complete nonlinear equations with minimum effort and for permitting an easy check on the derivation. The technique selected for this development is based on the principle of virtual work and uses standard orthogonal matrix notation to condense the bookkeeping problems in the expansion of these equations.

#### STATE OF THE ART

Several analytical techniques have been developed for predicting the blade airloads, blade response, and performance of rotors in forward flight. Early developments were based on the airloads analyses for autogyros by Glauert and Lock (Refs 1 through 5). In order to solve the airloads equations analytically, many simplifying assumptions were

1.2 made. Included among these were the assumptions of uniform inflow through the rotor disc, steady incompressible unstalled flow over the rotor blades, elimination of the rotor reverse flow region, single-degree-of-freedom inelastic blades, and linearization of small motions. Various investigators modified the airloads analyses of Glauert and Lock to eliminate some of the aforementioned assumptions. Wheatley (Ref 6) extended the original analyses and Bailey (Refs 7 and 8) simplified the resulting equations and presented them in chart form that could be used for practical engineering calculations. Gessow and Myers (Ref 9) discuss these methods and their underlying assumptions in detail. Among other contributors were Hohenemser, Sissingh, Tapscott, Gustafson, Gessow, and Myers. A list of reports by these authors is presented in Reference 9. With the advent of the high-speed digital computer in the mid-1950's, numerical solutions could be obtained of the highly nonlinear blade airloads equations. This new powerful tool created a quantum jump in the analytical capability of the rotor analyst. Many numerical techniques and solutions to the rotor airloads problem have been formulated. Among the early contributors using numerical methods of solution are Gessow and Crim (Ref 10), Piziali and Chang (Refs 11 and 12), Brandt (Ref 13), Miller (Ref 14), Blankenship and Harvey (Ref 15), Berman (Ref 16), LaForge (Ref 17), Scully (Ref 18), and Duhon et al (Ref 19). More recently, contributions to the theoretical prediction of blade airloads, blade responses, and rotor performance have been made by Lemnios et al (Refs 20 and 21), Arcidiacono et al (Ref 22) and Landgrebe (Ref 23). This paper summarizes the methods developed in Refs 20 and 21, presents a comparison of these methods with available test data, describes their application to improve an existing rotor, and summarizes the results of an optimization study of a new rotor concept. DESIGN PROCEDURE Two basic tenets were prescribed and maintained throughout the development of the analyses reported herein. First, the analyses provide sufficient detail to describe blade behavior and rotor characteristics accurately. Second, the analyses are automated to minimize manual interfacing and data handling thereby minimizing the possibility of introducing inadvertent errors. A flow chart illustrating the rotor blade design process is shown in Figure 1. The design loop procedure is initiated by reading coordinates and material properties from prepared drawings. These data are used as input to computer programs that generate distributions of mass, center of gravity, moments of inertia, and stiffness. Usually, the number of radial stations and their location do not coincide with the corresponding input requirements of the aeroelasticity program. Consequently, the data are submitted to a program that automatically reconfigures all the distributions to prescribed radial stations. The reconfiguration analysis redistributes the input data so that the integrated blade physical characteristics remain unchanged. Included among these physical characteristics are blade mass, first feathering moment, first flapping moment, feathering moment of inertia, flapping moment of inertia, and cross product of inertia between flapping and feathering. The reconfiguration analysis outputs the distributions automatically on punched cards, magnetic tape, or a storage disk. This analysis is especially useful and time-saving on blades that have non-uniform mass distributions. Output from the reconfiguration program is used directly as input to the acroelasticity program and to an analysis for blade statics and dynamics. This latter analysis evaluates static droop and uses standard eigenvalue procedures to calculate frequencies and mode shapes of flatwise, edgewise, and torsion modes. Frequencies and modes are automatically prepared as input to the blade aeroelasticity analysis. The aeroelasticity analysis is derived in detail in Reference 20 and expanded in Reference 21. The coupled equations of motion include six response modes and two control modes for a fully articulated rotor system as shown in Figure 2. The response modes are treated as normal modes and include blade pitching, blade lagging, blade flapping, blade flapwise bending, blade elastic twisting, and control flap pitching. Output from the aeroelastic analysis includes transient time histories of the six modes in response to a step input to any of the modes selected by the user. The transient responses are a measure of the coupled system stability and can be used to predict stability boundaries. Additionally, the aeroelastic analysis outputs the steady state time histories, load distributions, and angle of attack distributions of the fully coupled, nonlinear equations of motion. The load distributions are integrated to obtain hub forces, hub moments, and main rotor horsepower. A trim analysis program automatically changes rotor control inputs until the rotor is trimmed to a flight condition preselected by the user. The load distributions of the trimmed rotor are harmonically analyzed and are automatically input to a bending moment analysis that calculates root shears, blade deflections, blade slopes, and blade bending moments. The aeroelastic analysis and the trim analysis are described in more detail in the following sections. Results of the above analyses are examined and evaluated by the rotor blade design team and are used as a quide to modify the original design thereby closing the first design loop and initiating a second loop. The process is repeated until a satisfactory design is reached; usually three major iterations are sufficient.

#### AEROELASTICITY ANALYSIS

As mentioned previously, the derivation of the equations of motion and basic method of solution for the aeroelasticity analysis have been reported in detail in References 20 and 21. They are discussed in the present paper only to the extent that a general understanding of the method is developed.

The total energy of the system must satisfy a balance such that it can be applied to the Lagrangian equations of motion for the jth generalized coordinate as established in Reference 20.

$$fff\ddot{x}_{i}(\partial x_{i}/\partial s_{j})dm + \partial u_{1}/\partial s_{j} + \partial u_{2}/\partial \dot{s}_{j} = ff(\partial x_{i}/\partial s_{j})dp_{i}$$
 (1)

The strain energy function is of the form

$$U_1 = U_1(s_i) \tag{2}$$

and the dissipative energy function is of the form

$$U_2 = U_2(\dot{s}_i) \tag{3}$$

As in Reference 20, the coordinates of a point on the flap are given by

$$X_2 = Vt + \Psi[E_1 + Z\{E_2 + B[P + \Phi_{tw} \cap O_X(Q + E_3 + \Delta E_2)]\}]$$
 (4)

and those for a point on the blades are given by

$$X_1 = Vt + \Psi[E_1 + Z\{E_2 + B[P + \Phi_{tw} \cap O_X(O + E_1)]\}]$$
 (5)

where the transformation matrices are defined in Appendix I. The evaluation of  $\ddot{x}_i (\partial x_i/\partial s_j)$  will produce all of the inertial terms for the jth degree of freedom. The order of multiplication is immaterial because the resultant product is a scalar. If a matrix operator  $D_1$  is defined as in Appendix I, the inertial terms for the equations of motion can be evaluated from the following integral

$$\int D_1 X^T dm X = inertial and centrifugal moments$$
 (6)

In performing this matrix multiplication, advantage is taken of the orthogonality of the matrices.

The strain energy potential function,  $\mathrm{U}_1$ , for the system is obtained by assuming that each rigid body degree of freedom contains a torsion spring whose output torque is proportional to the angular deflection (or slope) of the mode under consideration. Spring rates for the two types of control systems are also included in order that accurate control loads can be calculated. The dissipation function,  $\mathrm{U}_2$ , is obtained by similarly assuming a viscous damper whose output torque is proportional to angular velocity. The elastic modes are included in these two functions by using the natural frequency, generalized mass, and structural damping associated with each mode. The moments from the two functions,  $\mathrm{U}_1$  and  $\mathrm{U}_2$ , are evaluated by operating on them with the two matrix operators,  $\mathrm{D}_1$  and  $\mathrm{D}_2$  that are defined in Appendix I.

$$D_1U_1 + D_2U_2 = \text{spring and damper moments}$$
 (7)

The surface forces acting on the rotor blade and flap consist of the aero-dynamic forces and moments which can be separated into steady and unsteady components. The generalized forces for Eq (1) are given by moments which account for the appropriate boundary conditions in each mode. The generalized forces can be written in terms of the virtual work done by the external forces.

$$Q_{j} = \int \int \frac{\partial x_{i}}{\partial s_{j}} dp_{i} = \frac{dW_{2}}{ds_{j}}$$
 (8)

Eq (8) is not in a convenient form for use with the equilibrium equations, because it is written in terms of pressures rather than the more common aerodynamic lift, drag, pitching moment, and hinge moment distributions along the blade and flap.

The generalized forces can be expressed in terms of these aerodynamic parameters by using strip theory to apply the distributed loads along the blade and allowing the blade to undergo a virtual displacement in each mode. When this is done, expressions for each  $Q_{\frac{1}{2}}$  can be written as follows:

$$Q_{\zeta} = \int [L \varphi_{\psi} + D \varphi_{\psi}] (r - e_{1}) dr \qquad Q_{q_{1}} = \int [L \varphi_{(0} + O_{x} - \phi_{y})] + D \varphi_{(0} + O_{x} - \phi_{y}) dr$$

$$Q_{\delta} = \int [L \varphi_{\psi} - D \varphi_{\psi}] (r - e_{2}) dr \qquad Q_{0} = \int M_{0} \phi_{tw} dr$$

$$Q_{0} = \int M_{0} dr \qquad Q_{\delta} = \int M_{\delta} dr \qquad (9)$$

Figures 3, 4, and 5 illustrate the blade airfoil cross-section geometry, the section aerodynamic force, moment, and velocity vectors, and the blade generalized forces.

The present analysis describes the behavior of articulated rotors with inboard pitch control input, with flap control input, or with dual control input. The feathering input is given by

$$\Theta_{\rm in} = \Theta_{\rm c} - \Theta_{\rm 1s} \$_{\psi} - \Theta_{\rm 1c} \$_{\psi} - \Theta_{\rm 2s} \$_{2\psi} - \Theta_{\rm 2c} \$_{2\psi} + \kappa_{\rm OB} B + \kappa_{\rm OC} C$$
 (10)

and the flap input is given by

$$\delta_{\mathbf{in}} = \delta_{\mathbf{c}} + \delta_{\mathbf{1s}} \mathbf{\$}_{\psi} + \delta_{\mathbf{1c}} \mathbf{¢}_{\psi} + \delta_{\mathbf{2s}} \mathbf{\$}_{2\psi} + \delta_{\mathbf{2c}} \mathbf{¢}_{2\psi} + K_{\delta\beta} \beta + K_{\delta0} 0 + \delta_{\zeta} \zeta$$
 (11)

In Equations (10) and (11), the various azimuthal coefficients correspond to collective, cyclic, and second harmonic inputs; the constants correspond to the mechanical feedback couplings among the modes.

Because of the response modes and control inputs included in the analysis, the equations can be used to analyze articulated rotors controlled directly with pitch horns, articulated rotors controlled aerodynamically with flaps, articulated rotors with dual controls, hingeless rotors controlled by pitch horns, hingeless rotors controlled by flaps, or hingeless rotors with dual controls.

#### METHOD OF SOLUTION

Generally, nonlinear response equations are solved by numerical methods, for computer adaptation. Most often, these solutions use a forward integration approach, which requires a knowledge of initial conditions in order to precipitate the solution (initial value problem). However, experience has shown that an estimate of the initial conditions can be so far from the steady-state solution of a multi-mode analysis that it is not possible to achieve stability or that it takes many iterations to get to the stable solution.

The method used for solving the above nonlinear equations is an expanded version of the method described by Berman for a two-degree-of-freedom system (Ref 16). This method is generally applicable to sets of nonlinear differential equations with coefficients that are arbitrary functions of a single independent variable; in this case, the variable is time. A brief description of the procedures is outlined below.

Initially, the set of nonlinear equations is linearized by making the usual small angle assumptions and neglecting terms higher than first order. The aerodynamic forcing functions are linearized only with respect to local angle of attack. Linearized aerodynamic force and moment derivatives are tabulated versus Mach number for the normal flow region, for the reverse-flow region, and for a narrow band around the reverse-flow circle corresponding to low dynamic pressures. The linearized differential equations are then written as a set of finite difference equations which are solved via the integrating matrix operator developed by Berman.

The direct application of an integrating matrix operator to the linearized equations written in matrix form yields two important results. First, the completely coupled transient response of the linearized system to an initial disturbance is calculated. The initial disturbance can be a displacement or a velocity step input in any single mode or any combination of modes. Transient responses are calculated for a prespecified time interval (usually 20 rotor revolutions) in order to evaluate the stability characteristics of the coupled system. Second, the response matrices of the system are obtained which depend only on the coefficients of the linearized equations. When the coefficients and the forcing functions are periodic, the response matrices are modified by the end conditions to yield periodic responses directly, without carrying the calculations through more than one cycle. Thus, the initial value approach for obtaining linearized transient solutions is transformed into a boundary value approach for obtaining nonlinearized periodic (steady-state) solutions.

The response to the complete set of nonlinear equations is obtained through the use of the above-mentioned periodic response matrices. The methodology is better understood by beginning with the general description of the linear solutions. The left-hand side of the equation is composed of the linear inertia matrix  $[I_L]$  and the linear airload matrix  $[L_L]$ , the sum of which is set equal to a forcing function matrix [F]. This leads to the following equation:

$$([I_L] + [L_L]) = [F]$$
 (12)

The left-hand side of Eq (12) includes functions of the displacements, velocities, and accelerations of the various response modes. Through the techniques reported in Ref 20, the left-hand side can be made functions of accelerations only so that Eq (12) can be rewritten in terms of the acceleration matrix [X], as shown in Eq (13).

$$[A] [\ddot{X}] = [F] \tag{13}$$

The periodic response matrix is defined by

$$\{P\} \equiv [A^{-1}] \tag{14}$$

where the matrices [A] and [P] are matrix operators. Eq (13) becomes

These matrices are integrated by the methods in Reference 16 to give velocities and displacements of the linear equations. These linear solutions are subsequently used as initial inputs for the iterative procedures in order to obtain nonlinear solutions.

The nonlinear equations can be similarly represented with nonlinear inertias  $[\textbf{I}_{NL}]$ , and nonlinear airloads  $[\textbf{L}_{NL}]$ , and set equal to the nonlinear forcing function  $[\textbf{F}_{NL}]$  as in the following equation:

$$([I_{NL}] + [L_{NL}]) = [F_{NL}]$$

$$(16)$$

This equation can be rewritten

$$0 = [F_{NL}] - ([I_{NL}] + [L_{NL}])$$
 (17)

If the linear inertias and loads are added to both sides, we get Eq (18)

$$([I_L] + [L_L]) = ([I_L] + [L_L]) - ([I_{NL}] + [L_{NL}]) + [F_{NL}]$$
 (18)

Using the definitions in Eqs (12) through (15), we can write the accelerations as follows:

$$\ddot{[X]} = [P]\{([I_L] + [I_L]) - ([I_{NL}] + [I_{NL}]) + [F_{NL}]\}$$
(19)

Equation (19) represents a complete set of nonlinear equations operated on by linear response matrices. The included linear effects are self cancelling by definition from Eq (18). The numerical solutions of the acceleration matrix [X] represent solutions for each response mode at every azimuth position.

Because the right-hand side of Eq (19) has terms which include modal velocities, displacements, and accelerations, it is solved by iterative methods. Subscripts are added to Eq (19) to indicate the successive iterations. The iterative nonlinear response equation is defined as

$$\ddot{x}_{k} = [P] \{ ([I_{L}] + [L_{L}])_{k-1} - ([I_{NL}] + [L_{NL}])_{k-1} + [F_{NL}]_{k-1} \}$$
(20)

where

k = present iteration count
k-1 = previous iteration count

and k has the range (1  $\leq k \leq N) with \ N$  being the number of iterations required for convergence.

The initial responses, i.e., displacements, velocities, and accelerations, are determined from the linear solution of Eq (15). These are substituted into the right-hand side of Eq (20) to generate the nonlinear matrices which are used to determine the second set of accelerations. In general, these new accelerations are not identical to those that were input on the right-hand side because the linear solution does not have the same accelerations as the nonlinear solutions. Therefore, these accelerations are integrated to obtain new displacements and velocities which are reinserted into the right-hand side to obtain a third solution to the accelerations. These substitutions are repeated until the kth responses coincide with the (k-1)th responses to within specified tolerances. The last set is the converged responses.

#### COMPUTER PROGRAM

A computer program for the aeroelastic analysis was written in FORTRAN IV language for an IBM 360/40 digital computer with a Disk Operating System, and a 132,000 word storage capacity. The computer program can handle a rotor blade with 16 radial stations which are used to describe nonuniform radial distributions of chord, twist, airfoil section, mass, moment of inertia, chordwise center of gravity, bending mode, twisting mode, elastic axis, and aerodynamic center. Sixty azimuth positions can be evaluated for each of the six response modes. Aerodynamic force and moment coefficient wind tunnel data are tabulated for each of 3 airfoil sections at 5 Mach numbers and 49 angles of attack, one of the airfoil sections has coefficient data tabulated for 5 flap settings in addition to the Mach number and angle-of-attack ranges. Unsteady airfoil characteristics are estimated by force and moment derivatives based on Theodorsen theory. Variable inflow calculations are treated as a subroutine in the overall aeroelastic methods.

Figure 6 illustrates schematically the basic program logic for computation of the rotor airloads and responses. Data for the specific rotor are input, and rotor inertias and linear aerodynamic terms are calculated. Pertinent information is stored on the disk for intermediate storage. The coefficients of the linearized equations are calculated and integrated to form the [A] matrix. This matrix is inverted to form the periodic response matrix which is stored on the disk for use in all subsequent solutions. The initial value matrix is determined, and the transient responses are obtained as a user option. The boundary conditions are generated and also stored on the disk.

The above portion of the computer program is known as Part I and must be run for every flight condition or for any change in the physical properties of the rotor. Part II of the computer program contains the input requirements for the controls (i.e.,

1-6

pitch horn and/or flap controls) and calculates the steady-state responses of the fully coupled nonlinear equations. Therefore, it must be run for each new control setting. Using the data stored on the disk and the input for the controls, the linear forcing functions are now calculated, and the linear steady-state responses are determined. The local angles of attack are computed from these linear responses, and the local airloads are determined from blade element theory. The nonlinear forcing functions are obtained using these airloads, and new blade responses are computed. A trivariate interpolation scheme is used to obtain aerodynamic data for conditions that are intermediate to tabulated values. The iterative process of computing nonlinear airloads and nonlinear responses is cycled until convergence is reached between successive iterations.

Rotor performance is calculated from the converged airloads by integrating the various airload distributions radially and azimuthally over the disk. The expressions used to evaluate rotor thrust, torque, drag, and side force are given below.

$$T_{s} = \frac{b}{2\pi} \int_{0}^{2\pi} \left\{ \frac{R}{1} \left( L \varphi_{\phi_{v}} - D \varphi_{\phi_{v}} \right) \varphi_{\beta} dr d\psi \right\}$$
 (21)

$$Q_{s} = \frac{b}{2\pi} \int_{0}^{2\pi} \left\{ \frac{R}{1} \left( L \right) + D \right\}_{v} + D \psi_{v} \right) r dr d\psi$$
 (22)

$$H_{s} = \frac{b}{2\pi} \int_{0}^{2\pi} e_{1}^{R} [(L *_{\phi_{V}} + D *_{\phi_{V}}) *_{\psi} - (L *_{\phi_{V}} - D *_{\phi_{V}}) *_{\beta} *_{\psi}] dr d\psi$$
 (23)

$$Y_{S} = -\frac{b}{2\pi} \int_{0}^{2\pi} \left[ \left( L \hat{s}_{\phi_{V}} + D \hat{\phi}_{V} \right) \hat{\phi}_{\psi} + \left( L \hat{\phi}_{\phi_{V}} - D \hat{s}_{\phi_{V}} \right) \hat{s}_{\beta} \hat{s}_{\psi} \right] dr d\psi$$
 (24)

Blade responses are printed in the form of radial and azimuthal distributions of angles of attack, airloads normal to the shaft plane, airloads in the shaft plane, feathering moments, torque, Mach numbers, and critical Mach number ratios. Time histories of the six coupled blade responses, pitch horn control loads, and flap rod control loads are also output. Standard harmonic analysis techniques automatically resolve the waveforms of angles of attack, airloads, moments, and blade responses to yield the harmonic content of each parameter. Angle-of-attack distributions are also interpolated automatically to locate radial stations corresponding to integer values in angles of attack. These latter results are used to generate angle-of-attack contour plots.

#### ROTOR TRIM

The aeroelastic loads analysis produces a set of forces, blade responses, and rotor performance for a specific set of control inputs. However, the forces produced are not necessarily the forces required for trim at the particular flight conditions. The method for achieving the proper control inputs to obtain the necessary trim forces is called the trim program.

Figure 7 is a flow chart that shows how the trim program is used. Several cases are run with control inputs which are estimated to produce trim, and rotor forces and moments are generated. These initial forces and moments are used to produce derivatives which are then used to estimate new control inputs. This procedure is iterated until a trimmed condition is achieved.

As seen in Figure 8, a rotor system with a single control (either pitch horn or aerodynamic flap), has a unique combination of collective and cyclic control inputs which will generate the required hub trim forces at a specified speed. However, a rotor system with dual controls has several combinations of collective and cyclic inputs which will produce the same trim forces. Thus, optimizing the control inputs for a dual control rotor involves a procedure which requires several times as many trimmed computer runs as is normally required for a conventional single control rotor. The trim conditions resulting from this optimization procedure generate a response surface that is used to estimate control inputs which maximize performance and/or minimize blade dynamic response. To complete the procedure, these estimated optimum control inputs are used to generate new trimmed optimum flight conditions for the rotor.

#### BENDING MOMENTS

The linear partial differential equations of motion for combined flatwise bending, edgewise bending, and torsion of twisted nonuniform rotor blades are derived in detail by Houbolt and Brooks in Reference 24 and are extended in Reference 20 to include the Coriolis force in the edgewise direction due to flatwise bending. The structural dynamics equations are solved in Reference 20 through the use of harmonic analysis techniques. The aerodynamic forces and solutions are assumed to be periodic of the form:

Substitution of Equations (25) into the structural dynamics equations results in a set of three simultaneous differential equations in terms of  $Y_n$ ,  $Z_n$ , and  $\Phi_n$  and their derivatives. The resulting equations are integrated numerically from the blade tip via integrating matrices. Boundary conditions are applied at the blade root that are appropriate to articulated, hingeless, or teetering rotors. Thus, the equations are transformed into a set of finite difference equations in  $Y_n$ ,  $Z_n$ , and  $\Phi_n$  only which can be solved to generate harmonic response matrices similar to those of Reference 25. Post-multiplication of these response matrices by the airloads harmonics obtained in the aeroelastics analysis yields the blade bending moment responses and the vibratory hub shears. Details of the method of solution are presented in Reference 20.

#### APPLICATIONS

In order to evaluate the accuracy of the analyses, comparisons were made with flight test data obtained on an H-34 fully instrumented rotor blade (Reference 26). The data are compared for level flight conditions at advance ratios of 0.129, 0.228, and 0.299; all comparisons are shown in Reference 20. Representative comparisons of aerodynamic loading, pitch horn loads, flatwise bending moments, and radial bending moment peaks for two advance ratios are shown in Figures 9 through 12, respectively.

As seen in Figure 9, the predicted aerodynamic loads on the advancing blade contain impulses that are more severe than those measured on the blade. The impulsive behavior results from the use of a simplified variable inflow theory that contains two trailing vortices in the wake pattern located at the tip and at 35 percent of blade radius. The predicted airload comparisons can be improved by using a more detailed variable inflow analysis that includes more trailing vortices, a shed vortex wake, and a finite core diameter for the vortices.

Peak values of pitch horn loads in Figure 10 compare well with the flight loads; an approximate 30 degree pitch phase shift occurs between the two curves. The analytical results can be used confidently to predict linkage design loads for pitch horn rotor systems.

Figure 11 shows azimuthal variations in flatwise bending moments for the critical radial station. The steady bending moments were subtracted from the total measured values because of inconsistencies between flight. The correlation between test and analysis indicates that the analytical results can be used to predict realistic blade design loads and to estimate blade life. Figure 12 presents double amplitude values of flatwise bending moments along the blade radius. These curves contain the values obtained from Figure 11 as well as values at other stations reported in Reference 26. The calculated bending moments correlate well with test data.

As a result of the confidence gained in the aeroelasticity and bending moment analyses, a rotor modification program was initiated to improve the performance and flying qualities of the U. S. Navy H-2 helicopter at a gross weight of 12,500 lbs. The design improvement program resulted in the 101 Rotor. Among the key objectives of the 101 Rotor design were a guaranteed 40 knot increase in retreating blade stall margin and a guaranteed increase in blade life to 3000 hours. These improvements were to be achieved with no change in blade geometry, airfoil section, or blade solidity.

The rotor design loop described in Figure 1 was used to optimize the blade for the 101 Rotor. Predictions for the 101 Rotor configuration indicated an increase in retreating blade stall margin of 40 knots. Retrimming the servo flap, and thereby redistributing airloads along the span, provides 22 knots of stall relief. Increasing rotor speed from 287 rpm to 300 rpm adds another 18 knots to the stall margin. A 3000 hour blade life is achieved by reducing the peak spanwise bending moments by 15 percent via mass balance changes to detune blade response. Blade mass balance selection was also influenced by comparing criteria for blade stability on the aeroelasticity program with test criteria.

Flight test measurements on the 101 Rotor are reported in Reference 27. Figures 13 and 14 present test curves obtained with the 101 Rotor compared with those of the standard rotor and show that the optimization efforts gave test results better than predicted. The level flight retreating blade stall boundary exceeded the stall boundary quarantees by an average of 8 knots. The blade peak bending moments were 20 to 25 percent lower than those of the standard configuration. Blade lives calculated from flight strain measurements exceeded the 3000 hour quarantee.

Another recent application of the aeroelastic loads and trim analysis was a parametric investigation of the Controllable Twist Rotor system. This rotor system consists of torsionally soft blades with dual controls. The controls consist of conventional pitch horn linkages at the inboard end and an aerodynamic control flap at the outboard end.

A general blade configuration was selected for study of both a conventional pitch horn rotor system and a CTR system. The blades selected had identical chords and radii. A broad parametric evaluation of the CTR system was performed with a four-bladed configuration to determine the effects of blade torsional frequency, blade built-in twist, flap configuration, flap size, and flap location on rotor performance and blade dynamics. Based on the results of these analyses, a nearly optimum CTR configuration was chosen for further analysis and for comparison with the conventional rotor system which was analyzed with four, five, and six blades.

Full details of the optimization study and the comparison are presented in Reference 21. The comparison between the CTR and the conventional rotor was made by sizing the rotor systems to perform a nominal utility helicopter mission. The results indicate that the dual control CTR is smaller and has fewer blades than the comparable DCR (Direct Control Rotor) thereby resulting in a lighter, smaller vehicle with lower power requirement. The following tables summarize the comparisons between the two rotors.

TABLE
ESTIMATED PERFORMANCE PARAMETERS AND WEIGHT BREAKDOWNS FOR THE
MISSION ANALYSES COMPARING THE 4-BLADED CTR TO A 6-BLADED DCR

Aircraft Performance Parameters		
	DCR	CTR
Rotor Diameter (ft) Solidity (0) Drag Area (ft) Installed Power (Mil SL 59°F Rating) (HP) Disk Loading (lb/ft²) Blade Loading (lb/ft²) Main Rotor Tip Speed, $\Omega$ R (fps) Hover Power Requirements (OGE, 4000 ft, 95°F) Hover Power Available (4000 ft, 95°F)	48 .156 28.3 4520 7.56 48.5 661 2000 3200	3500 7.56 72.8 661
Statistical Weight Breakdown		
	DCR	CTR
Structures, Rotor, Transmission Equipment Twinned Powerplants & Installation Fuel Fuel System Mission Load	6120 850 2050 3380 300 1000	2460 210 1000
Takeoff Gross Weight	13700	11500

\* Includes CTR Weight Penalties; 80 1b for Flaps and Mass Balance; 170 1b for Duplicate Control.

#### CONCLUSIONS

- 1. A comprehensive nonlinear aeroelastic loads analysis has been developed to evaluate articulated or hingeless rotors with single or dual controls.
- 2. A fully automated trim/control optimization analysis has been developed and integrated with the aeroelasticity analysis.
- 3. An automated blade bending moment analysis has been developed and integrated with the aeroelasticity analysis.
- 4. Automated input data preparation analyses have been developed and integrated with the aeroelasticity analysis.
- 5. The analyses have been correlated successfully with available test data from a fully instrumented blade.
- 6. The analyses have been applied successfully to improve the performance and dynamic behavior of a rotor system in service use.
- 7. The analyses have been used to evaluate and optimize a new dual control rotor system.

- 1. Glauert, H., "A General Theory of the Autogiro", British Aeronautical Research Council R and M Number 1111, 1926.
- Lock, C.N.H., "Further Development of Autogiro Theory", British Aeronautical Research Council R and M Number 1127, 1927.
- Glauert, H., "Lift and Torque of an Autogiro on the Ground", British Aeronautical Research Council R and M Number 1131, 1927.
- Glauert, H., "On the Vertical Ascent of a Helicopter", British Aeronautical Research Council R and M Number 1132, 1927.
- Glauert, H. and Lock, C.N.H., "A Summary of the Experimental and Theoretical Investigations of the Characteristics of an Autogiro", British Aeronautical Research Council R and M Number 1162, 1928.
- Wheatley, J.B., "An Aerodynamic Analysis of the Autogiro Rotor With a Comparison Between Calculated and Experimental Results", NACA Report 487, 1934.
- 7. Bailey, F.J., Jr., "A Simplified Theoretical Method of Determining the Characteristics of a Lifting Rotor in Forward Flight", NACA Report 716, 1941.
- 8. Bailey, F.J., Jr. and Gustafson, F.B., "Charts for Estimation of the Characteristics of a Helicopter Rotor in Forward Flight. I Profile Drag-Lift Ratio for Untwisted Rectangular Blades", NACA ACR L4H07. 1944.
- Gessow, A. and Myers, G.C., Jr., "Aerodynamics of the Helicopter", USA, The Macmillan Co., 1952, Chapters 8 and 9.
- 10. Gessow, A. and Crim, A.D., "A Method for Studying the Transient Blade-Flapping Behavior of Lifting Rotors at Extreme Operating Conditions", NACA Technical Note 3366, January 1955.
- 11. Piziali, R.A., "A Method for Predicting the Aerodynamic Loads and Dynamic Response of Rotor Blades", U. S. Army AVLABS Technical Report 65-74, January 1966.
- 12. Chang, T.T., "A Method for Predicting the Trim Constants of a Single-Rotor Helicopter and the Rotor Blade Loadings and Responses", U. S. Army AVLABS Technical Report 67-71, November 1967.
- 13. Brandt, D.E., "Investigation of Rotor Blade Dynamics During Equilibrium Transition Flight", M.S. Thesis, M.I.T., June 1962.
- 14. Miller, R.H., "Theoretical Determination of Rotor Blade Harmonic Airloads", M.I.T. Aeroelastic and Structures Research Laboratory TR 107-2, August 1964.
- 15. Blankenship, B.L. and Harvey, K.W., "A Digital Analysis for Helicopter Performance and Rotor Blade Bending Moments", Journal of the American Helicopter Society, October 1962.
- 16. Berman, A., "High Speed Rotor Research Part II, A New Approach to the Solution of Rotor Blade Dynamic Problems", Journal of the American Helicopter Society, July 1965.
- 17. LaForge, S.V., "Effects of Blade Stall on Helicopter Rotor Blade Bending and Torsional Loads", Hughes Tool Company, Aircraft Division, HTC AD 64-8, May 1965.
- Scully, M.P., "Approximate Solutions for Computing Helicopter Harmonic Airloads", M.I.T. Aeroelastic and Structures Research Laboratory TR 123-2, December 1965.
- 19. Duhon, J.M. et al, "Computer Flight Testing of Rotorcraft", Journal of the American Helicopter Society, October 1965.
- Lemnios, A.Z., Smith, A.F. and Berman, A., "The Aeroelastic Behavior of Rotary Wings in Forward Flight, Volumes I through IV", Kaman Aerospace Corporation Report R-585, January 1968.
- Lemnios, A.Z. and Smith, A.F., "An Analytical Evaluation of the Controllable Twist Rotor Performance and Dynamic Behavior", U. S. Army AMRDL Technical Report 72-16, May 1972.
- Arcidiacono, P.J. et al, "Prediction of Rotor Instability at High Forward Speeds, Volumes I-V", U. S. Army AVLABS Technical Report 68-18A-E, February 1969.
- 23. Landgrebe, A.J., "An Analytical and Experimental Investigation of Helicopter Rotor Hover Performance and Wake Geometry Characteristics", U. S. Army AMRDL Technical Report 71-24, June 1971.
- 24. Houbolt, J.C. and Brooks, G.W., "Differential Equations of Motion for Combined Flapwise Bending, Chordwise Bending and Torsion of Twisted Nonuniform Rotor Blades", NACA Report 1346, 1958.

- 25. Berman, A., "Response Matrix Method of Rotor Blade Analysis", Journal of the Aeronautical Sciences, Volume 23, Number 2, February 1956.
- Scheiman, J., "A Tabulation of Helicopter Rotor Blade Differential Pressures, Stresses, and Motions as Measured in Flight", NASA Technical Memorandum TM X-952, March 1964.
- 27. Parkinson, R. and DuFresne, D.A., "Phase I Navy Preliminary Evaluation of the Model HH-2D/101 Rotor System", Naval Air Test Center Technical Report FT-108R-71, 30 December 1971.

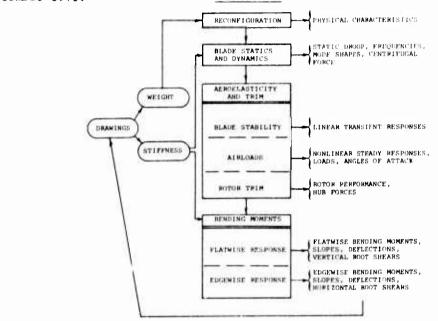


Figure 1. Rotor Blade Design Loop

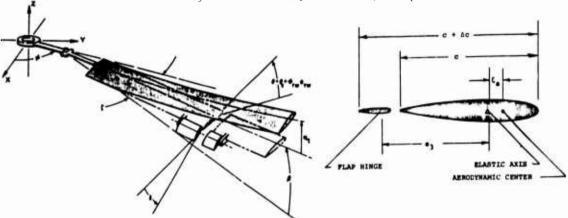


Figure 2. Displacements for Aeroelasticity Figure 3. Blade and Flap Geometry Analysis

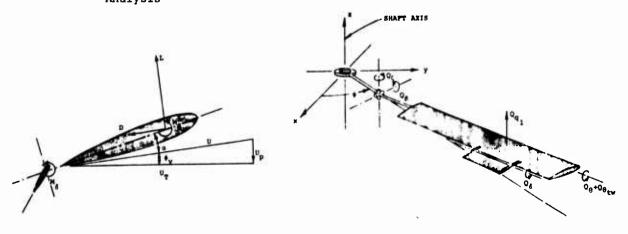


Figure 4. Airfoil Section Indicating
Aerodynamic Force and
Velocity Vectors

Figure 5. Generalized Forces of the Rotor System

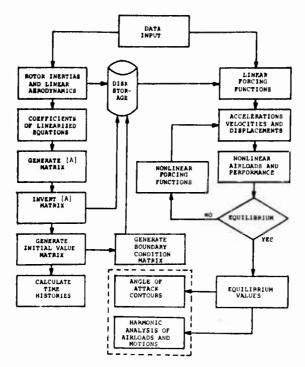


Figure 6. Flow Chart of the Aeroelasticity Computer Program

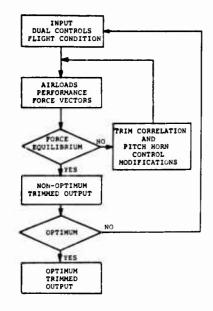


Figure 7. Flow Chart for Trim/Control Optimization Analysis

#### TRIM ANALYSIS

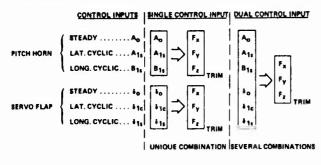


Figure 8. Control Input Combinations for Rotor Trim

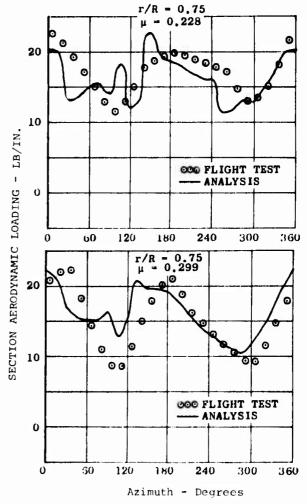


Figure 9. Section Aerodynamic Loading Comparisons

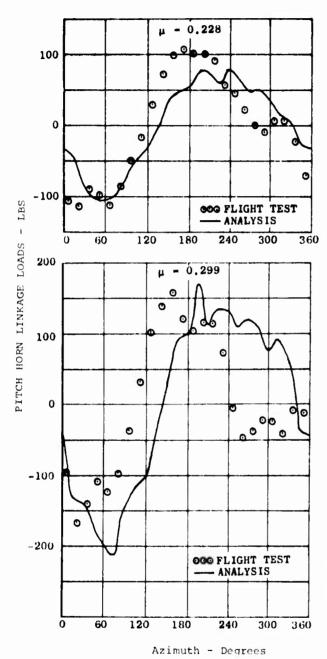


Figure 10. Pitch Horn Linkage Vibratory Load Comparisons

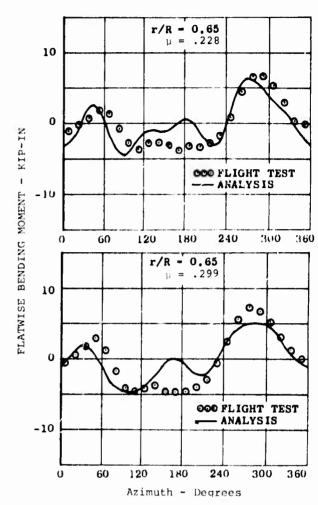


Figure 11. Flatwise Bending Moment Comparisons

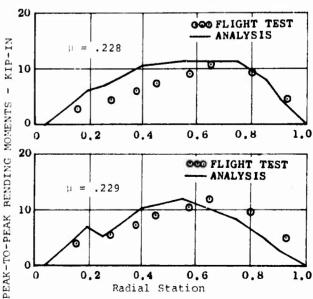
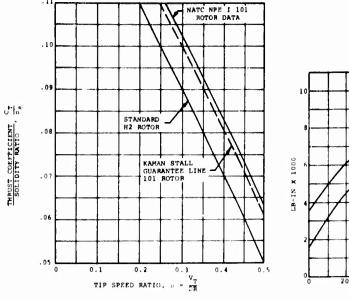
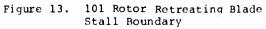


Figure 12. Flatwise Vibratory Bending Moments as a Function of Radial Station





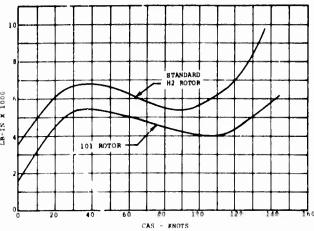


Figure 14. 101 Rotor Blade Flatwise Bending Moment

## APPENDIX I MATRIX TRANSFORMATIONS

The following is a list of matrix transformations used in Eqs (4) and (5).

$$\Xi_{1} = \begin{bmatrix} 0 \\ \xi_{1} \\ 0 \end{bmatrix} \quad \Xi_{2} = \begin{bmatrix} 0 \\ \xi_{2} \\ 0 \end{bmatrix} \quad E_{1} = \begin{bmatrix} e_{1} \\ 0 \\ 0 \end{bmatrix}$$

$$E_{2} = \begin{bmatrix} e_{2} - e_{1} \\ 0 \\ 0 \end{bmatrix} \quad E_{3} = \begin{bmatrix} 0 \\ e_{3} \\ 0 \end{bmatrix} \quad V = \begin{bmatrix} -V_{X} \\ V_{Y} \\ V_{Z} \end{bmatrix}$$

$$Q = \begin{bmatrix} -\Delta r \\ 0 \\ q_{1} \phi_{0} \end{bmatrix} \qquad P = \begin{bmatrix} r - e_{2} \\ 0 \\ 0 \end{bmatrix}$$

$$Q = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \psi_{\theta} & -\psi_{\theta} \\ 0 & \psi_{\theta} & \psi_{\theta} \end{bmatrix} \qquad Q_{X} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \psi_{\theta} & -\psi_{\theta} \\ 0 & \psi_{\theta} & \psi_{\theta} \end{bmatrix}$$

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \psi_{\theta} & -\psi_{\theta} \\ 0 & \psi_{\theta} & \psi_{\theta} \end{bmatrix} \qquad Q_{X} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \psi_{\theta} & -\psi_{\theta} \\ 0 & \psi_{\theta} & \psi_{\theta} \end{bmatrix}$$

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \psi_{\theta} & -\psi_{\theta} \\ 0 & \psi_{\theta} & \psi_{\theta} \end{bmatrix} \qquad Q_{X} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \psi_{\theta} & -\psi_{\theta} \\ 0 & \psi_{\theta} & \psi_{\theta} \end{bmatrix}$$

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \psi_{\theta} & -\psi_{\theta} \\ 0 & \psi_{\theta} & \psi_{\theta} \end{bmatrix} \qquad Q_{X} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \psi_{\theta} & -\psi_{\theta} \\ 0 & \psi_{\theta} & -\psi_{\theta} \end{bmatrix}$$

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \psi_{\theta} & -\psi_{\theta} \\ 0 & \psi_{\theta} & \psi_{\theta} \end{bmatrix} \qquad Q_{X} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & \psi_{\theta} & -\psi_{\theta} \\ 0 & \psi_{\theta} & -\psi_{\theta} \\ 0 & \psi_{\theta} & -\psi_{\theta} \end{bmatrix}$$

$$Q = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & \psi_{\theta} & -\psi_{\theta} \\ 0$$

where  $\psi_u = \text{cosine } u$  $\psi_u = \text{sine } u$ 

Note that the rotation matrices are orthogonal so that the transpose of each is equal to its inverse.

$$\psi^{T} = \psi^{-1}$$
,  $z^{T} = z^{-1}$ ,  $B^{T} = B^{-1}$ ,  $O^{T} = O^{-1}$ ,  $O_{\mathbf{x}}^{T} = O_{\mathbf{x}}^{-1}$ ,  $O_{\mathsf{tw}}^{T} = O_{\mathsf{tw}}^{-1}$ ,  $\Lambda^{T} = \Lambda^{-1}$ 

Furthermore, each rotation matrix depends only on one generalized coordinate.

The matrix operators  $D_1$  and  $D_2$  are defined by

$$D_{1} = \begin{bmatrix} \frac{\partial}{\partial \zeta} \\ \frac{\partial}{\partial \beta} \\ \frac{\partial}{\partial \theta} \\ \frac{\partial}{\partial q_{1}} \\ \frac{\partial}{\partial \delta} \\ \frac{\partial}{\partial \theta_{tw}} \end{bmatrix} \qquad D_{2} = \begin{bmatrix} \frac{\partial}{\partial \dot{\zeta}} \\ \frac{\partial}{\partial \dot{\beta}} \\ \frac{\partial}{\partial \dot{\theta}} \\ \frac{\partial}{\partial \dot{\theta}} \\ \frac{\partial}{\partial \dot{\theta}} \\ \frac{\partial}{\partial \dot{\delta}} \\ \frac{\partial}{\partial \dot{\theta}} \\ \frac{\partial}{\partial$$

#### **CURRENT LOADS TECHNOLOGY FOR HELICOPTER ROTORS**

by

Richard Gabel
Manager, Structures Staff
The Boeing Company
Vertol Division
P. O. Box 16858
Philadelphia, Pennsylvania 19142

#### SUMMARY

Prediction of fatigue design loads is essential for proper sizing of helicopter rotor systems. Boeing-Vertol has developed the C-60 rotor loads computer program. It incorporates the effects of airfoil section geometry, compressibility, stall, three-dimensional flow, unsteady aerodynamics, and nonuniform inflow to provide reliable rotor loads for steady-state flight conditions even into the blade stall region. Rotor loads predictions are compared with actual flight test data from Boeing CH-47 and Model 347 helicopters. An approach to component sizing is presented in which a fatigue design loads histogram is constructed using calculated steady-state flight loads and empirically-determined maneuver loads. Current efforts to improve rotor loads predictions through incorporation of fully coupled lag-pitch-flap routines, simulation of control system dynamics, and development of maneuver loads programs are discussed.

#### SYMBOLS

E	Modulus of Elasticity	V	Local Air Velocity, Aircraft Forward
I	Moment of Inertia		Speed
G	Shear Modulus	ī,	Vortex Circulation Strength
J	Polar Moment of Incrtia	$\mathrm{C}_{\mathrm{T}}$	Coefficient of Thrust
$c_{L}$	Airfoil Lift Coefficient (Subscripts 2D	13	Rotor Solidity, Standard Deviation
-	and 3D Indicate 2-Dimensional and	μ	Advance Ratio
	3-Dimensional Coefficient; No	$R_{\mathbf{D}}$	Density Altitude
	Subscript Indicates 21)	Car	Modal Frequency
В. Е.	Blade Element	GW	Gross Weight
a	Angle of Attack	CG	Center of Gravity
Λ	Sweep or Yaw Angle	Nr	Rotor Blade Rotational Frequency
52	Rotor Angular Velocity	RPM	Revolutions Per Minute
$\Delta \Gamma$	Average Adjacent Blade Lift	r/R	Blade Station, Percentage Measured
R	Bay Length		Outboard from Rotational Center
D	Air Density	INLB	Moment or Torque Inch Pounds

#### INTRODUCTION

This paper discusses helicopter rotor sizing from a loads viewpoint. The rotor loads prediction methods currently in use at The Boeing Company's Vertol Division have been developed to fit into the iterative process of rotor sizing. Of course, rotor loads depend in part on the elastic and mass properties of the rotor, and therefore an estimate of the rotor properties is required to generate loads (hence the iteration process). For this paper, the word rotor means blades, hub, and upper controls.

Helicopter systems are developed to perform a given mission or series of missions. It is beyond the scope of this paper to discuss in detail the trade studies conducted to determine basic helicopter configuration. Indeed, such a discussion would be a paper in itself. It is sufficient to say that mission requirements expressed in such terms as payload, range, hover performance, fuel reserve, and maneuver capability provide the basis for configuration selection. Thrust requirements (dictated by hover performance at design gross weights and by maneuver capability over the operating envelope) and airfoil characteristics are used to determine blade radius, blade chord, rotor RPM and horsepower required. Blade radius is constrained further by disc loading criteria, blade chord by flying qualities criteria, rotor RPM (i.e. tip speed) by noise criteria, and so on.

The net result of these preliminary trades is a configuration selection for which basic helicopter geometry and helicopter system target weights are identified. Rotor blade radius and chord, hub geometry, control system configuration, and target weights for each system are defined. The discussion of rotor loads, for the purpose of this paper, presumes the configuration selection has been completed. The success of rotor system component sizing (i.e. structural reliability at minimum weight) depends strongly on the proper definition of rotor system design loads.

#### **AEROELASTIC ROTOR LOADS COMPUTER PROGRAM**

Boeing-Vertol has developed an aeroelastic rotor loads program, the C-60 program, which has proven adequate for helicopter loads predictions. Typical program running time on the IBM 360 computer is 10 minutes. Program C-60 calculates rotor blade flapwise, chordwise, and torsional deflections and loads together with rotor performance, control system forces, and vibratory hub loads. Articulated and hingeless rotors with from 2 to 9 blades and low twist may be analyzed. The analysis is limited to calculations involving steady-state flight at constant rotor tipspeeds. The blades may be of arbitrary planform, twist, and radial variation in airfoil section.

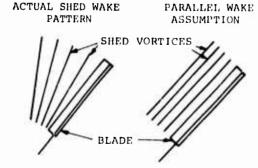
The analysis considers coupled flapwise-torsion deflections and uncoupled chordwise deflections of the rotor blades. The blade is represented by 20 lumped masses interconnected in series by elastic elements. Boundary conditions for either articulated or hingeless rotors are applied and the solution obtained by expanding the variables in a 10-harmonic Fourier series. Rotor blade idealization is shown in Figure 1.

#### ACTUAL BLADE THE BLADE HAS: SMALL, NONLINEAR TWIST VARIABLE SHEAR CENTER & VERTICAL NEUTRAL AXIS NONLINEAR MASS & ELASTIC PROPERTIES VARIABLE PLANFORM & CROSS SECTION APPROXIMATION EACH SECTION HAS: NO TWIST BLADE SECTION CONSTANT SHEAR BOUNDARIES CENTER & VERTICAL NEUTRAL AXIS NO ELASTIC DEFLEC-TION BUT IS CONNECTED TO THE ADJACENT SECTION THROUGH EQUIVALENT STIFFNESS CONSTANT PLANFORM & CROSS SECTION EQUIVALENT SYSTEM APPLIED AIRLOADS CONSTANT EI & GJ ELASTIC BAY BLADE SECTION mmmmm **BOUNDARIES** PITCH AXIS MASS BAY

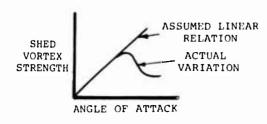
Figure 1. Comparison of Actual Blade and Idealized Blade

EQUIVALENT MASS

Airload calculations include the effects of airfoil section geometry, compressibility, stall, 3-dimensional flow, unsteady aerodynamics, and nonuniform inflow. Static airfoil tables are used to account for compressibility, static stall, and airfoil shape. The unsteady aerodynamic loads are calculated by modifying the static loads resulting from the airfoil tables to include Theordorsen's shed-wake function modified by the assumptions shown in Figure 2, dynamic stall effects based on oscillating airfoil data (Figure 3), and yawed flow across the blade (Figure 4).



# LINEAR RELATION BETWEEN VORTEX STRENGTH AND ANGLE OF ATTACK



#### REDUCED FREQUENCY

USES THE INSTANEOUS RELATIVE VELOCITY AND ASSUMES ONLY 1/REV BLADE MOTION IS PRESENT

Figure 2. Shed Wake Assumptions

The nonuniform inflow calculations are based on a tip and root vortex trailed from each blade (Figure 5). Through an iterative technique, each trailed vortex is made compatible with the calculated blade lift distribution; and the lift distribution is compatible with the nonuniform downwash field. The effect of downwash on lift is shown in Figure 6. The vortex wake is assumed to be rigid and to drift relative to the hub with a constant resultant velocity composed of thrust-induced uniform downwash and the speed of the aircraft.

A flow diagram of this analysis is shown in Figure 7. The solution for the nonlinear aerodynamic loads and the coupled flap-pitch blade response is performed in series. Up to 10 iterations between

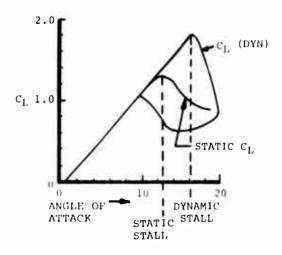


Figure 3. The Dynamic CL Calculation in Stall Is Dependent on the Dynamic Stall Delay

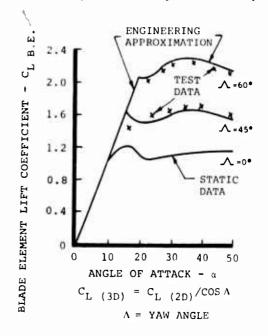
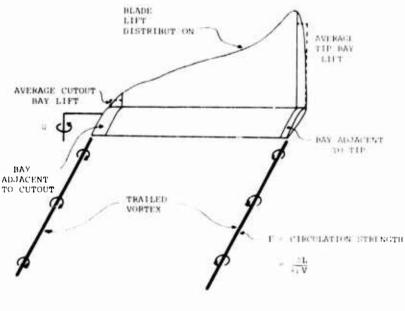


Figure 4. Engineering Approximation Accounting for Yawed Flow

the airloads and blade response are used to obtain the final solution. An iterative solution is used to account for the nonlinear coupling between the blade deflections and airloads that result from airfoil stall and compressibility. Iteration techniques are also used to obtain compatibility between the airloads, downwash, and vortex strength and to obtain a match with a specified rotor thrust. The following is a brief outline of the computer procedure.

The C-60 program is started by calculating initial deflections and defining boundary conditions from input (i.e. collective and cyclic pitch and root flap deflections). These inputs are either known (as in the case of a model test) or are obtained from an aerodynamic trim analysis. The trim analysis calculates the rotor trim (i.e. aircraft angle of attack, thrust, collective pitch, cyclic pitch, and blade root flap angle) by considering aircraft gross weight, center of gravity, fuselage drag, rigid blade properties, quasilinear static airfoil characteristics, nonuniform downwash, forward speed, and rotor speed to determine the airloads required to maintain the free flight aircraft in equilibrium for a steady-state flight condition.

Next, the rotor-induced velocities are calculated to provide a downwash field for each blade. First, uniform downwash is determined either from input or a simple calculation. If only uniform downwash is required, the program exits from the downwash routine and proceeds to the airload routine. If nonuniform downwash is required, a complex iteration loop is initiated. The downwash field resulting from this routine is used throughout the program with no updates or modification.



AVERAGE ADJACENT BLADE LIFT

BAY LINGTH

AIR DINSITY

LOCAL VELOCITY

Figure 5. Calculation of Trailed Vortex Strength

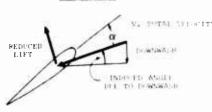
field is calculated, the rigid blade deflections (initial deflections), nonuniform downwash, and nonlinear aerodynamic coefficients are then combined to calculate the airloads. Here again, a thrust routine iteration is provided to guarantee a thrust match if desired. Following the airload routine, the aerodynamic vertical force and pitching moment are harmonically analyzed and used as forcing functions to calculate the coupled flap-pitch dynamic response of the blade. Since the forcing loads are nonlinear functions of the forced response (due to stall and compressibility), iteration between aerodynamic loads and the blade dynamic response is required to provide feedback. To perform the iteration, the most recently calculated coupled flap-pitch deflections are substituted back into the aerodynamic analysis, the aerodynamic forcing functions are evaluated again, and the coupled flap-pitch response analysis repeated. The iteration is continued until a specific number of iterations (usually 10) has been completed. The number of iterations specified

After the downwash

should be sufficient to insure a converged solution.

NO DOWNWASH SECTION LIFT (PROPORTIONAL TO AIRFOIL ANGL V? AND OCI M4 RELATIVE VELOCITY

WITH DOWNWASH



THE DOWNWASH FROM THE TRAILED VORTICES REDUCES THE ANGLE OF ATTACK. THE REDICTION IS ANGLE OF ATTACK REDICES THE LIFT (LIFT DEFICIENCY) AND HENCE REDICES THE STRENGTH OF THE TRAILED VORTEX SINCE THE TRAILED VORTEX STRENGTH IS PROPORTIONAL TO THE BLADE SECTION

Figure 6. Effect of Downwash

After the coupled flap-pitch airload iteration is complete, the Coriolis force is calculated from the coupled flap-pitch deflec-

tions and added to the tangential aerodynamic forces for the lag forcing function. The uncoupled lag response is calculated in the same manner as the coupled flappitch response; however, no iterations are performed. The influence of lag deflections on airloads and the coupled flap-pitch response is assumed to be negligible.

Next, blade radial forces are calculated by considering blade shortening resulting from flap and lag deflections; pitch link loads are calculated by determining the blade system pitching moment and all loads are resolved from the rotor disc system

into the blade system. Finally, fixed and rotating system hub and lower control loads are evaluated by combining the root shears and moments with the system geometry.

As the flow diagram shows, the aeroelastic rotor analysis is basically an aerodynamic analysis coupled by iteration to a dynamic analysis.

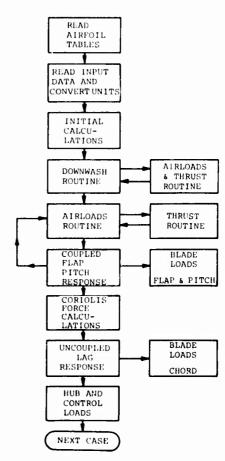


Figure 7. C-60 Flow Diagram

The process of developing a rotor loads prediction capability goes hand-in-hand with a continual effort to correlate predictions with actual test results. The question is: How good are the predicted rotor system loads? Several examples of loads correlation results have been selected to provide visibility on this question. Flight test data from three of Boeing-Vertol's tandem rotor helicopters and one run of wind tunnel data is shown. The CH-47C helicopter has two rotors with three constant-chord constant-thickness blades of steel-fiberglass construction; the Advanced Geometry Blade CH-47 has two rotors with three tapered-chord tapered-thickness blades of fiberglass construction, and the Model 347 helicopter has two rotors with four CH-47C blades. Pitch link loads shown are from the CH-47C aft rotor at  $\mu=0.306$  and  $C_{\rm T}/\sigma$  of 0.1147 (well into blade stall) and from a 14-foot-diameter wind tunnel model. Table I summarizes the flight conditions and test data shown in the correlation plots. All the test data shown is for steady-state flight conditions.

	Number	Blade					CG			Com	onent
Source	of				Density Altitude		Location	Max CT/	Max	Pitch	
CH-47C	3	30	8.3	39000	7000	230	5 Aft	0.1147	0.306	х	Х
Wind Tunnel	4	7	BVWT 102-44	-	-	1023	-	0.062	0.350	х	
CH-47 (AGB)	3	30	272	46000	6000	245	4 Aft	0.0890	0.299		x
Model 347	4	30	313	42000	1600	220	6 Aft	0.0873	0.373		х

#### Pitch Link Loads

The pitch link, which reacts blade pitching moments, introduces flight loads into the control system. Historically, high controlsystem loads associated with blade stall have limited the helicopter flight envelope. Alternating pitch-link loads typically exhibit a rather consistent increase with increasing airspeed until blade stall is reached. Then pitch-link loads and hence control system loads may exhibit rather sharp increases in magnitude including significant harmonic content from multiples of the once-per-rotor revolution frequency. Figure 8 shows measured CH-47C alternating

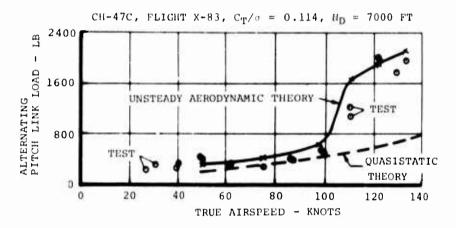


Figure 8. Comparison of Test and Analytical Pitch Link Loads for an Airspeed Sweep

pitch link loads as a function of airspeed. The C-60 load predictions prior to the incorporation of compressibility, stall, dynamic stall histeresis, and three-dimensional flow is labeled quasistatic theory and fails to predict pitch link

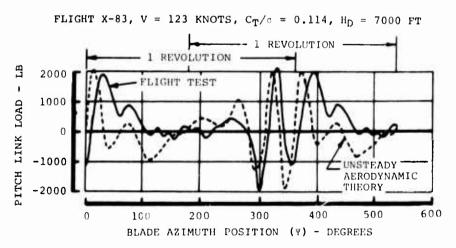


Figure 9. Pitch Link Load Waveform Correlation Using Unsteady Aerodynamic Theory

loads above 100 knots. With the incorporation of unsteady aerodynamic theory, the improvement in pitch link load correlation is dramatic. Not only are the loads below stall predicted well, but good agreement with stall flutter loads is obtained, and the inception of blade stall at 97 knots is accurately predicted. Figure 9 shows the predicted and measured pitch link waveform at 123 knots. The capability to predict the stall flutter load spike magnitude and location in the fourth quadrant is a significant advancement in rotor loads technology.

Figure 10 shows a pitch link load waveform taken from data recorded in the Boeing-Vertol Wind Tunnel in August 1972. The model was a 14-foot-diameter four-bladed mach scaled rotor and the data was recorded for  $C_{\rm T}/\sigma$  = 0.062 and  $\mu$  = 0.350. The C-60 prediction, scaled down from full size to model dimensions, shows exceptional correlation in both waveform and magnitude.

#### **Blade Flap Bending Moments**

Rotor blade alternating (one-half peak to peak) flap bending moments for the CH-47C, CH-47 (AGB), and the Model 347 are shown in Figures 11, 12, and 13. The test data variation is indicated by a bar, and the average or most representative alternating load recorded is indicated by a circle. C-60 predictions using both uniform and nonuniform downwash routines are included in the high speed cases shown. The use of nonuniform downwash in

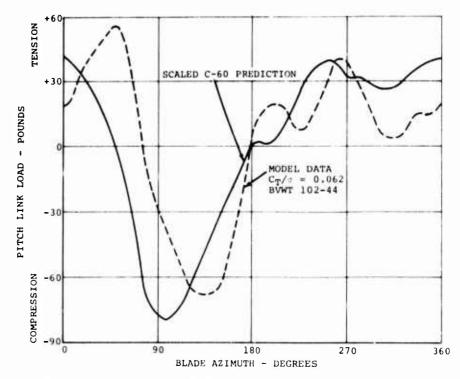


Figure 10. Comparison of Wind Tunnel Pitch Link Load and the Scaled C-60 Prediction

the analysis more closely predicts the lift distribtuion along the blade as a function of azimuth and also introduces blade tip vortices lying in or near the disc plane. For single rotor helicopters (or forward rotors on tandem helicopters), blade loads are increased by the impulse from the intersection of a blade with the vortex of a preceding blade. In tandems, the aft-rotor loads can be increased by the intersection of aft blades with vortices trailing downstream from the forward rotor. Vortex strikes or near strikes tend to impart impulse loads to the blade exciting the higher flap bending modes. These modes are lightly damped and therefore contribute significantly to the flap bending moment when vortex interference is involved. For the CH-47C blade, the second  $(4.75\,\Omega)$ , third  $(7.90\,\Omega)$ , and

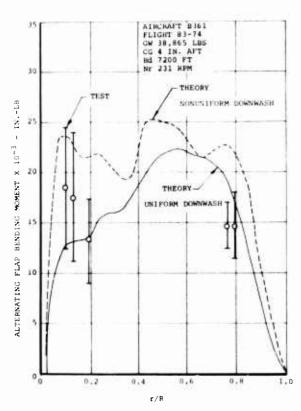


Figure 11. CH-47C Aft Rotor, Predicted and Measured Alternating Flap Bending Moment at V = 123 Knots

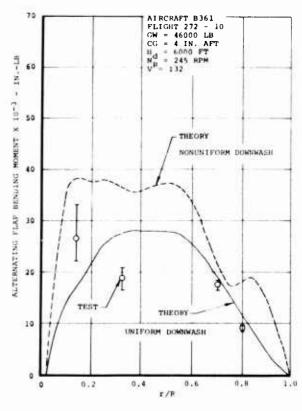


Figure 12. CH-47C With Advanced-Geometry Blades, Aft Rotor Predicted and Measured Alternating Flap Bending Moment at 132 Knots

fourth (12.19 Ω) flexible modes all peak at 10 to 13 and 80 to 85 percent radius. This indicates that flap bending moments, when operating with significant vortex disturbance, would have a large inboard and outboard peak. The effect of the fourth flexible mode is not included because the C-60 program capability is limited to 10 harmonics.

#### COMPONENT SIZING

During flight qualification of an actual helicopter, an extensive array of loads are collected. These loads cover the full range of helicopter gross weight, center-of-gravity location, airspeed, altitude, rotor RPM, cyclic trim and helicopter maneuvers. Flight loads/stresses are recorded on many components (approximately 150 stress parameters were recorded during the CH-47C Chinook stress and motion survey) including rotor blade flap bending, chord bending, torsion and absolute spar stresses; rotating control system stresses on hubs, pitch links, drive scissors, and swashplate lugs; stationary control system stresses on actuators and linkages; rotor shaft bending moment and torsion; drive system torsion; and airframe stresses at selected locations. A helicopter mission (fatigue) profile (Table II) is derived from the mission requirements and used with the loads above to define the structural flight envelope. A further step is the establishment of component retirement lives.

This process is fine for aircraft qualification, but during initial component sizing measured flight loads obviously do not exist. Even so, the definition of a design fatigue load histogram relating to the helicopter fatigue profile is now feasible. It can presently be done by a mixture of pure analysis (C-60 rotor loads program), wind tunnel test data, and empirical use of flight test data from existing helicopters. In the future, it may be possible to predict envelope loads completely by analytical means.

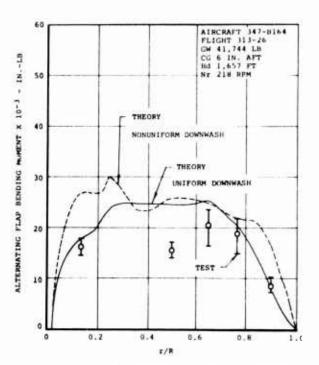


Figure 13. Model 347 With CH-47C Blades, Aft Rotor Predicted and Measured Alternating Flap Bending Moment at V = 152 Knots

TABLE II.

TYPICAL HELICOPTER FATIGUE PROFILE\*\*

CONDITION	PERCENT OF OCCURRENCES*
Ground Conditions Take Off Steady Hovering Turns Hovering Hover Control Reversals Sideward Flight Rearward Flight Landing Approach Forward Flight 20 Percent VH 40 Percent VH 50 Percent VH 70 Percent VH 70 Percent VH 80 Percent VH 90 Percent VH	1.0 (400) 16.0 (1000) (1000) 1.0 0.5 (500) 4.0 2.0 2.0 5.0 18.0 16.0 16.0
115 Percent V <sub>H</sub> Climb, Takeoff Power Climb, Full Power Partial Power Descent Turns	1.0 3.0 4.0 (500) 5.0
Controls Reversals Pull Up Power to Autorotation Autorotation to Power Steady Autorotation Autorotation	(800) (250) (40) (40) 1.0 0.4
Autorotation Control Rev Autorotation Landing Autorotation Pull Up Ground-Air-Ground Power Drive	0.3 0.3 (40) (100) 2.5

- Bracketed numbers are occurrences per 100 flight hours
- \*\* Altitude, gross weight, and center of gravity splits not shown for convenience

As an illustration of the semiempirical process, consider the following pitch link discussion which is presented as an example of component sizing.

Figure 14 shows CH-47C flight test data which, for the example cited, represents a configuration similar to the new design. The CH-47C maneuver data is used as the basis for maneuver loads for the new design. For convenience, only pull-ups, turns, and control reversals are shown. Figure 15 shows the calculated (C-60) steady state pitch link load for the new helicopter as a function of airspeed for the design gross weight. The mission required airspeed VH, the power limit, and stall inception (indicated by the calculated waveform) are shown for reference. The maneuver loads shown in Figure 15 are related empirically to the actual CH-47C data shown in Figure 14. Care must be exercised in the determination of the maneuver loads to properly account for penetration into blade stall and for configuration differences such as delta three. The empirical relationships consider relative  $C_{\mathrm{T}}/\sigma$  and may account for as many splits of maneuver severity (i.e. load factor or bank angle) as desired. The pitch link fatigue design loads histogram (Figure 16) is now constructed using the loads data and the frequency of occurrence specified in the mission profile.

The required pitch link size depends not only on loads but also on material fatigue properties. With the S-N curve shape of the structural material involved and the loads histogram, a life versus endurance limit curve, Figure 17, is constructed by setting the endurance limit at various load levels and using Miner's cumulative fatigue damage rule. Entering the life versus endurance limit curve at the required design life defines the required pitch link endurance limit expressed in load. This design load is used to size the part in conjunction with mean -3 o fatigue stress allowables. Conventional fatigue analyses accounting for stress concentrations, fretting, mean stresses, and secondary stresses are employed in component sizing. The mean -3 of allowable provides structural reliability for inherent scatter in material fatigue performance.

# EXPANDING PREDICTION CAPABILITY

Continued efforts are required to achieve a capability to predict rotor loads for an entire helicopter flight. Current efforts at Boeing-Vertol include an expansion of the C-60 rotor loads program, the development of an analog program with the capability to simulate control system dynamics, and the development of loads programs for transient flight conditions. Each of these efforts is discussed below.

#### C-70 Rotor Loads Program

The C-60 Rotor Loads Program has been

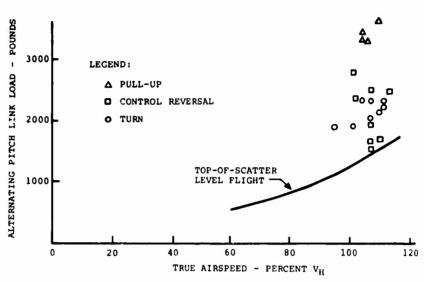


Figure 14. CH-47C Flight Test Data, Pitch Link Load versus True Airspeed

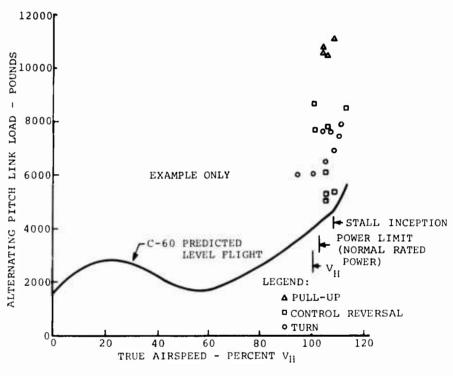


Figure 15. Fatigue Design Data, Pitch Link Load versus True Airspeed

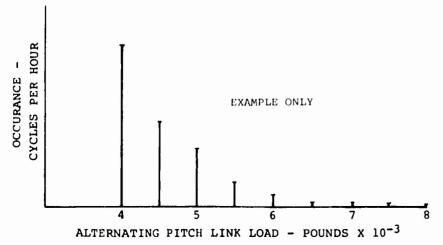


Figure 16. Pitch Link Fatigue Design Loads Histogram

expanded (and hence redesignated C-70) to allow the analysis of highly twisted propellers/rotors. Improvements, which should also benefit helicopter loads predictions to some degree, include the effect of large blade twist, shear center chordwise location, vertical neutral axis chordwise location, and fully coupled blade deflections.

For highly twisted blades, the principal axes in the chordwise and flapwise directions change their orientation relative to the rotorshaft at each spanwise location.

The C-60 analysis does not consider this principal axis

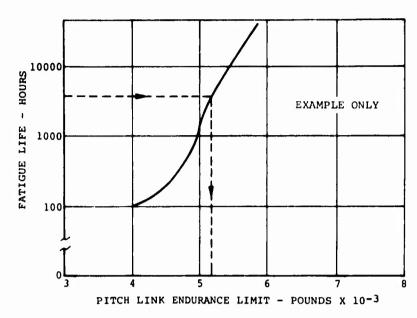


Figure 17. Pitch Link Fatigue Life versus Endurance Limit

rotation. To accurately calculate the blade deflections, including principal axis rotation, requires that either all calculations be performed in a local-axis system coincident with the local principal axis orientation or in a fixed-axis system with all the cross-inertia terms included. In either case, the analysis is best performed if the chordwise and flapwise deflections are coupled. In addition, since the chordwise mass center is not coincident with the shear center, coupling with torsion will also result. Therefore, to properly analyze rotors with large twist, a coupled chordwise-flapwise-torsion analysis is required.

The present C-60 rotor analysis assumes that the pitch axis, vertical neutral axis, and shear center are coincident. This assumption is fairly accurate when applied to long, slender rotor blades with spar-type construction. However, recent developments in composite materials,

new construction techniques, and the larger chord-to-radius ratio of propellers have made the difference between the pitch axis and shear center more important. Control input (cyclic and collective), as well as flexible control input resulting from control loads, are input about the pitch axis while elastic twist resulting from blade torsional loads pitches about the shear center. When the difference between the pitch axis and shear center is significant, both axis systems must be taken into account when calculating the rotor loads.

The C-70 prediction of flap bending moment on the Model 160 performance model is hover is shown in Figure 18 along with the wind tunnel data. The wind tunnel model is a 5.5 foot diameter 1/10 scale full span tilt rotor model. The test data was collected with a 3-degrees-of-cyclic and 10-degrees-of-collective control input. The blade has 36-degree twist over the airfoil section. The result shown speaks for itself.

#### **CBO Analog Analysis**

A CBO analog analysis program is being developed to account for the effects of control system dynamics on loads. Parametric studies indicate that control system (i.e., pitch link) loads are affected by control system stiffness. This is not really surprising because the control system may be thought of as a rotor torsional spring connected

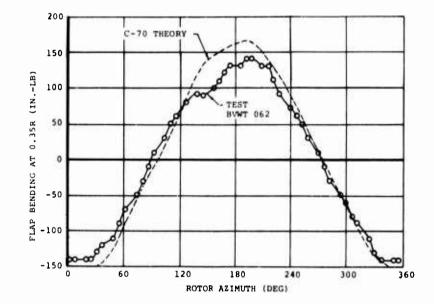


Figure 18. Model 160 Performance Model, Predicted and Measured Flap Bending Waveform in Hover for 3° Cyclic and 10° Collective at 2300 RPM

in series with a torsionally flexible rotor blade. It is also apparent that control system damping can be effective in delaying the onset or reducing the amplitude of pitch link load increases triggered by stall flutter. The CBO analog analysis is a six-degree-of-freedom single rotor dynamical analysis for four rotating blades which are forced with airloads that feature unsteady and stall delay aerodynamics. The rotor blades have individual first flexible torsional degrees of freedom with each blade connected to the swashplate through its own torsional spring. Distributed or lumped masses may be used to represent the swashplate under which three or four parallel spring-damper lower supports may be arbitrarily positioned about the azimuth. The system features individual blade aerodynamics with dynamic and aerodynamic swashplate-blade coupling.

The airloads are determined by a rapid blade radial airload integration on the analog computer. The loads are stored and applied as the forcing function to a set of simultaneous differential equations. Updating of the aerodynamic forcing function occurs every four degrees of rotation. The loading technique allows solutions to proceed more quickly than other programs. The analog receives control signals from the digital computer which also stores the basic airfoil data. Figure 19 is a schematic of the mathematical model.

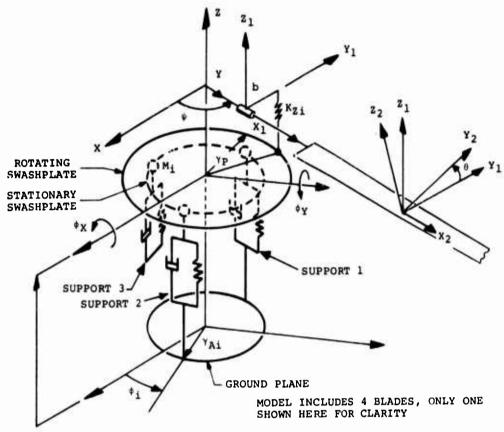


Figure 19. Four-Bladed Rotor Pitch Damping Model

Model 347 flight test data for the lower supports is shown with analog results in Figure 20. In this case, the analog blades were disconnected and the corresponding flight test pitch link waveform was used as input data. Note the correlation is good in amplitude but needs improvement in phase angle.

#### Maneuver Loads

An area of extreme interest in rotor loads technology is the prediction of transient loads related to maneuvering flight. An example of development efforts in this area is Program L-32 which can be used to predict vertical blade loads resulting from maneuvers and uniform gusts. L-32, using a companion program as a subroutine, calculates flap mode shapes of an articulated or rigid rotor. It utilizes these mode shapes to calculate the flap bending blade response and hub loads during steady-state conditions and during transient conditions following pilot control changes of collective, cyclic, and/or a gust. The collective and cyclic changes can be put in together or separately as a step or ramp function. The gust input is uniform starting and ending as a step function but may come from an arbitrary direction. The airloads include steady plus three harmonics, determined by linear aerodynamic theory, and a tip vortex prescribed by input in both space and time. The effects of delta three and precone are included and rotors with up to six blades can be accommodated. Although L-32 is currently limited to vertical loads and does not account for the removal of control inputs, some reasonable correlation with measured CH-47B flight test data has been achieved as shown in Figure 21.

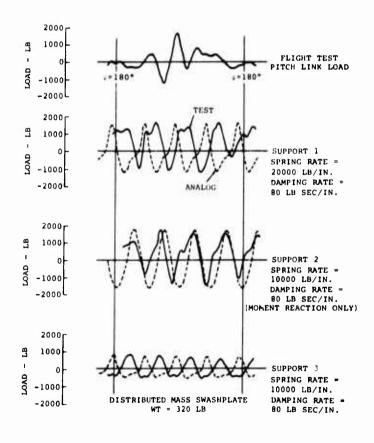


Figure 20. 347 Helicopter Aft Rotor Analog Correlation With Flight Test

#### FORWARD ROTOR WITH VORTEX

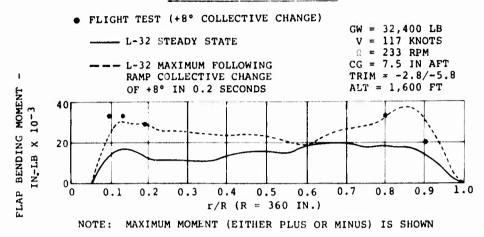


Figure 21. CH-47B Rotor Blade Flap Bending Moment

#### CONCLUSIONS

The capability for predicting helicopter rotor system loads for rotating components is well established for unstalled level-flight conditions. Recent developments such as the inclusion of unsteady aerodynamics in computer programs allows calculation of stalled pitch link loads. Research efforts expanding loads prediction technology to include lower control loads, to account for control system dynamics, to handle highly twisted prop/rotors, and to calculate transient loads during maneuvering flight are encouraging. They must be continued to achieve a capability for the analytical determination of total envelope loads. In the interim, empirical methods may be used with calculated level-flight loads to determine design loads for new helicopters.

#### BIBLIOGRAPHY

- 1. Theodorsen, T., GENERAL THEORY OF AERODYNAMIC INSTABILITY AND THE MECHANISM OF FLUTTER, NACA Report 496, National Advisory Committee for Aeronautics, Washington, D.C., 1935.
- 2. Tarzanin, F., PREDICTION OF CONTROL LOADS DUE TO BLADE STALL, 27th Annual National V/STOL Forum of the American Helicopter Society, Preprint No. 513, May 1971.
- 3. Alexander, H., Amos, A., Tarzanin, F., Taylor, R., V/STOL DYNAMICS AND AEROELASTIC ROTOR-AIRFRAME TECHNOLOGY, VOLUME II, DESCRIPTION AND CORRELATION OF NEW METHODOLOGIES, AFFDL-TR-72-40, May 1972.

REVIEW OF PAPERS 1 AND 2

by

A J Sobey
Principal Scientific Officer
Structures Department
Royal Aircraft Establishment
Farnborough
Hampshire
GU14 6TD
UK

A review may take several different forms - that of a book, for example, is a sales promotion exercise. In that light, I would like to applaud the two papers which it has been my pleasure to review and to declare that the first paper is an excellent account of the problem of rotor load determination. Those who wish to enter this difficult field could scarcely improve upon Dr Lemnios' paper with its extensive bibliography as an entree to the subject. Most of Dr Lemnios' paper is taken up with an account of the methodology which the rotor analyst performs and only a small amount of text is given up to other features so that the paper principally presents the results of several man-years of work which the Kaman Company have done in this field. Mr Gabel's paper is somewhat different in its broad conception. The author reports not one method, on which he asks to be judged, but an organic set of programmes progressively growing. Thus it is more candid in that it is continuously changing as a result of experience. Such updating would seem to be an essential part of any modern programme development.

A review may, however, like that of a TV programme, be critical and I will now pick up some points which have caught my eye as a structures man. Firstly, though, I should tread some delicate ground. The question of rotor load determination is one in which structural factors interplay with aerodynamic ones in such an interactive way that it is scarcely possible to separate the one effect from the other. Nevertheless, I will say very little about aerodynamic matters. Let us, then, examine some of the structural problems which occur in both programmes.

First of all there is the question of how to represent the blade. Now Dr Lemnios does not spell out in great detail how the dynamic model is to be established but he makes two very good points. The first is that the governing equations of motion are non-linear and secondly he uses the principle of virtual work to derive the governing equations of motion. He subsequently talks about blade modes. Presumably the governing equations of motion are simplified by the omission of certain forcing terms and the resulting free vibration equations are recovered and linearised to that some normal modes may be calculated. Matrix concepts are introduced into the analysis, so as to keep the book-keeping straight as he puts it, so presumably the modes are used as a basis for blade analysis. Now in contrast Mr Gabel's model of the blade is quite different. In his first figure you see his expose of the blade as a series of not disjoint but adjointed elements with particular interrelationships between them. His equations of motion are based upon this simplified model which are retained throughout the calculation. He does not use a modal approach. It would seem to me, particularly in view of the evidence from flight tests, that the harmonic analysis - the modal content - is very important and in view of the fact that the corroboration between calculation and experiment is principally in the lower order modes, it is perhaps in the inclusion of higher order harmonics that the next stage in modifying the dynamic model will take place.

If we have got modes, and let us assume that that is a really straightforward exercise, there is the question of how are we going to use them. One of the aspects here that disturbs se is that since cyclic pitch is essentially a finite amplitude excitation and since normal modes exist only for infinitesimal departures from steady coning, is the case for using normal modes in the helicopter rotor analysis as overwhelming as it is in the corresponding fixed wing case. Now if we are examining, let us say, the tail plane motion of a civil airliner, there is no question that dynamical equations for that component would be written down in terms of normal coordinates and we would expect the departures from the steady state condition to be small enough to allow that treatment, and for that to be almost linear. But is the corresponding situation in the rotary field as overwhelming? Is it, perhaps, the case — and I put this as a question to all of us who are looking at the general strategy of how to conduct the aeroelastic analysis — that we are better off using a modal approach, or using a continuous (or even discrete) system of equations.

I would like now to say something about the integration of the blade governing equations. Dr Lemnios makes the point that if we take some value of the azimuth, anticipate the initial values of the blade variables and integrate round one cycle and hope to return to those values, that this process may not work well. The forward integration process, euphemistically, may or may not converge. But we know that it is physically possible to demonstrate situations in which the motion is not periodic with basic period of one revolution but it is possible, as with the split tip-path plane condition for solutions with period other than one revolution to interfere with the ones that really interest us. I would like to suggest that we are unwise to demand of our computers that we march, and integrate around the azimuth and that this is the best way of solving the problem. It is an obvious way, but is it necessarily the best?

Now some years ago, if we were faced with the question of the stability of a system described by a system of non-linear differential equations we would be forced to do what Dr Lemnios does in his examination of the stability during a change of state from one position to another, that is to look at the linearized equations of motion about a particular point and test the stability in that region. But it is now possible due to the growth of so thods that are essentially very old in concept but very modern in application - namely that of Lyapunov - for stability of non-linear systems to be explored in a global sense. Now I put this out as an idea: are we not ripe, in the non-linear analysis field, for some sort of parallel to the Lyspunov stability analysis which will help us with the forced response problem? Dr Lemnios gives us ar account of the Berman integrating matrix method, which is a linear concept that derives from the Pears-Baker or Matrizant method, that relates the state vector at one azimuth to its neighbour by a transfer matrix. We may progress around the azimuth by more than one technique and the Berman method is one way of doing it. But the marching forward process is not wholly vindicated and I would like to see the non-linear aspects broken by some mathematical artefact rather than a numerical one. It is, in theory at least, possible to replace differential equations by finite difference ones and to set up a system of algebraic equations whose order will be high but the matrix sparse so that we can determine in a single calculation conditions on the blade.

I do not want to say very much about the tests of credibility of the programme which I will leave to my colleague Mr Piziali but I am impressed with the way in which both our speakers have shown the effectiveness of their programmes in action. But here I must strike a note of warning. You tend, like gardeners, to show only your best blooms. Thus in accepting that the evidence that has been adduced looks very good I would invite us to inspect a little more closely and this cannot be done in open forum.

Both our speakers have a similar difficulty in that they have been associated with programme development over a long period of time - it is man-years of work that we are talking about - and the question that goes through one's mind is: "What is the way shead?" Here Dr Lemnios makes no comment - there is no space in his paper to squeeze it in. But the preponderant part of Mr Gabel's paper is an account of the way in which his company has learned to adapt the C-60 programme to the chastening experiences of real life. I am delighted to see the way in which programme C-70 emerges after major surgery has taken place on C-60. In particular I note that the blade motion is described by fully coupled equations, a point I am sure Dr Lemnios will applaud since he started with such equations. Blade motion in which the lagging motion is a derived quantity that follows once you have basically set up the pitch-flap interaction does not seem to me to be right but its a good start. May be with the kind of blade on which Boeing Vertol have tested their programme, the structural decouplings in the blade allow the lag and flap-pitch separation, but it is significant that Mr Gabel has written in his paper that it is new blades, new methods of construction that force the use of a fully coupled system. Also to be applauded is the inclusion of the Tarzanin dynamic stall effects which creates the opportunity for making more penetrative examinations of that very difficult region at high forward speed.

Now we ought to ask the cardinal question "How good is it all?" - how useful is it to be able to predict loads at 1.0 g in level flight. It is unquestionably true that the loads that are currently calculated are not fatigue damaging. The first sentence of Mr Gabel's paper declares as objective the rotor fatigue life prediction and he bravely takes up the question of the way in which the present capability can be used to estimate fatigue life. Somewhere in the middle of Table II, listing flight conditions and designating the proportion of total flying life given to each flight condition, in the fatigue-damaging 5%. It is a very difficult exercise to read across from something which is non-fatigue damaging (an intermediate load) to a fatiguing (extreme) load. Now what Mr Gabel says can be represented this way. We have a two state situation. On the one hand a calculation for an existing design and flight observations on the aircraft in a variety of conditions, on the other calculations only for another design from which we make the corresponding projection - by simple proportion the schoolboy might say. But the credibility of that exercise is in doubt. It does depend on one design being close enough to those which have gone - like the motor car industry where this year's model looks very like last year's - so that you may iterate the design. But we are now considering revolutionary concepts in rotary-wing design. With these types of aircraft can we be so confident of the projections? If the end-product of all this effort is that the analyses that we do, however interesting they are academically and scholastically, are, in the end to be interpreted very loosely in design, one might ask "Is it really worth while? This is a rhetorical question for the answer is obviously yes, not that we all have a vested interest in wanting this intriguing and difficult activity to go on. The fact is that we cannot escape from doing so.

#### ROTOR AEROELASTIC SIMULATION - A REVIEW

bv

R.A. Piziali VIZEX, INC. 4524 Bailey Avenue Amherst, New York 14226 United States of America

SUMMARY

The comments of this review are directed toward the overall community effort to develop rotor aeroelastic computer simulations. The observation is made that, while over the past 10 to 12 years there has been a significant expansion in the scope of the predictive capability of rotor simulations, there has not been, in general, significant improvement in the correlation of the predicted results with the real world. Those aspects of the rotor simulations where it is believed future efforts should be focused are considered they are (1) the aerodynamic representations used, (2) the validation procedures used, and (3) the solution techniques used. Finally, the practical usefulness of the present type of rotor aeroelastic simulations to the rotor system designer is questioned.

#### 1. INTRODUCTION

The presentations by Mr. Lemnios and Mr. Gable describe two specific rotor airloads prediction methods which, I believe, are generally representative of the state-of-the-art in the United States today. Therefore, while the discussion presented herein is applicable to these two methods, my comments are really directed toward the overall community effort to develop and improve rotor airloads prediction methods (or as 1 prefer, Rotor Aeroelastic Simulations).

The objectives of rotor aeroelastic simulations are the structural dynamic response of the system and its performance. The structural response includes, e.g., the blade and hub stresses, control system loads, transmitted shears, etc. while the performance includes such information as the rotor lift, propulsive force, shaft moments, and power required.

The efforts to formulate large scale, detailed digital computer simulations of helicopter rotor systems began in the early 1960's (e.g. References 1, 2, and 3). These early efforts assumed the blade motions to be known and concentrated on developing the aerodynamic representations. As a degree of success was obtained with these first aerodynamic representations, elementary structural dynamics representations of the rotor blades were added to the simulations and the aerodynamic representation improved (e.g. Reference 4). Since then the scope of rotor aeroelastic simulations has expanded considerably to include all of the significant blade degrees of freedom and their couplings (linear and non-linear), the control system response, trim of the rotor/fuselage combination, maneuvers and transient response, free wake, stall and reverse flow aerodynamics, etc., etc. However, there is one striking observation—that is, in general the degree of correlation of the predicted results with measured results has not improved significantly, if at all, since those early simulation efforts. This is a general observation which can be made if one compares the correlations of the early work with more recent correlations. Admittedly exemptions can be presented but I reiterate that in general (i.e., for various configurations, operating conditions, etc.) the degree of correlation has not improved significantly.

Only as an example, I have presented in Figure 1 a comparison of the measured and predicted time history of airload at \( \mathbb{N} = 0.75 \) (extracted from Figure 11 of Reference 4) for the same case as Mr. Lemnios has presented in the lower half of his Figure 9 (also reproduced here in Figure 1). (This comparison is in no way intended to reflect on the simulation developed by Kaman but their result is convenient to illustrate the point.) Similar qualitative comparisons can be made using other recent results! It should be noted that the scales and units are different for these two comparisons but all that is intended here is to show that there is no significant difference in the degree of correlation attained. As further example of the degree of correlation which was being attained earlier I have presented here in Figures 2 and 3 results extracted from References 3 and 4, respectively. So as not to be misleading, I have also presented in Figure 4 another example of an early correlation (from Reference 4) where the agreement was not particularly good. As is still true today, the capability of the rotor aeroelastic simulations to predict the real-world results is not uniform over the range of applications (i.e., configurations and operating conditions).

Thus to reiterate--in the past 10 to 12 years while there has bound a significant expansion of the scope of the predictive capability of rotor aeroelastic insulations, there has been little significant improvement in the correlation of the predicted results with the real world. One factor which has contributed to the lack of improvement in the accuracy of the simulations is, I believe, the absence of adequate validation procedures. A good validation procedure should not simply compare the final results in total but should provide information as to the source of the discrepancies.

The following discussions and comments relative to the state-of-the-art will cover several aspects--they are,

. representations for rotor aeroelastic simulation

solution techniques

validation considerations

practical usefulness - applications.

#### 2. REPRESENTATIONS FOR ROTOR AEROELASTIC SIMULATION

The rotor aeroclastic simulation can be considered as being composed of two separate but interacting aspects—the structural dynamic representation and a representation of the system aerodynamics. Given the forces acting on the structural dynamic representation, it must be capable of predicting the dynamic response of the system. Similarly, given the motions of the blades and of the air relative to a common reference, the aerodynamic representation must be capable of adequately predicting the resulting aerodynamic forces. It is generally agreed that the latter aspect, aerodynamics, is by far the least developed.

#### Structural Dynamics

The "technology" of structural dynamic representation is not presently the limiting factor in rotor aeroelastic simulations. The only practical limitation relative to the structural representation is the analyst's willingness to wrestle with the resulting complexity in the equations of motion, and the resulting increase in the computational effort. Thus structural dynamic representations can be as detailed as you please. The structural dynamic representation described by Mr. Lemnios is an example of the detail and sophistication possible. Because the technology of structural dynamic representation is not the limiting aspect of rotor aeroelastic simulation, there is no need to consider it further, at least not for the time being.

#### Aerodynamics

The aerodynamic representation of the rotor blades can conceptually be subdivided into two aspects for the purposes of discussion and for developing a suitable model. This subdivision is based on cause and effect relationships. First, there is the "stimulus" or excitation which is cause of the airloads. It consists of the relative motion of the airfoil with respect to the air, i.e., the blade section angle-of-attack, motions, wake induced velocities, and gust velocities. The second aspect, i.e., the effect, is the pressure response on the surfaces of the airfoil due to the stimulus. This is controlled by the airfoil shape, the boundary layer response, and the near wake shed vorticity streaming from the trailing edge.

Within the stimulus aspect of the aerodynamic representation there is no particular difficulty in determining those contributions from the blade operating conditions and dynamic response. However, the induced velocity contribution is a problem area. This is the result of each blade passing over the trailing vortex wake of preceding blade passages. It has been referred to as the "cobblestone road of the helicopter". These induced velocities experienced by the rotor blade are a function of the time and spacial development of the rotor wake. The rotor trailing vortex wake determines the rotor blade airloads and at the same time is determined by them. If the rotor aeroelastic response is to be simulated, the stimulus to the force generating mechanism of the rotor must be accurately defined. While much effort has been directed to this problem (e.g., References 5 - 8) the representation of the rotor wake is still probably one of the weakest aspects of rotor aeroelastic simulations.

The pressure response at the surfaces of an airfoil, for a given stimulus is controlled by the airfoil shape, the boundary layer response, and the near-shed wake from the airfoil. Before proceeding, the functional dependence of the airfoil lift and moment pressure integrals on the airfoil "stimulus" should be recalled from the linearized potential theory for the unsteady motion of thin airfoils. The unsteady lift and pitching moment of an airfoil depend on only the first four components (and their time derivatives) of the Glauert Series (an infinite cosine series) expansion of the chordwise distribution of velocities normal to the airfoil surface. This velocity distribution represents the airfoil "stimulus" due to the relative blade/air motions. Thus the lift and pitching moment depend on only the first four "components" of the chordwise distribution of the stimulus. The first and second terms of this series representation of the chordwise distribution of normal velocities are, respectively, the chordwise uniform and linearly varying components. The third and fourth terms represent the next two higher ordered chordwise variations. The rigid body motions of the airfoil (i.e. pitching and plunging) can directly contribute only to these first two terms while the induced velocities can contribute to all long terms. (The more difficult problem--drag estimation--is affected by all terms.)

The question as to the range of angles-of-attack actually experienced by rotor blades in flight keeps re-occurring. The term "angle-of-attack" derives from, and is most appropriate to, steady airfoil operating conditions. In view of the preceding discussion it is noted that, for non-stationary airfoil motion, the angle-of-attack is represented by the chordwise constant "component" of the distribution (i.e. the mean value) of velocities normal to the chord. It cannot be determined from the velocity at any one point.

The stimulating environment experienced by an airfoil section of a rotor blade is quite complex. At each radial station, because of the rotor operating conditions and the structural dynamic response of the rotor blades, the airfoil section experiences a time varying angle-of-attack, rate of change of angle-of-attack, and plunging velocity which contribute only to the first two components of the stimulus. In addition it experiences a time varying induced velocity field which contributes to all four components of the

stimulus. These four components of the stimuli occur at all harmonics of the rotational speed--thus their sum results is a rather general but periodic time variation (for steady flight conditions).

Now let's consider the pressure response aspect of currently used aerodynamic representations. First it must be noted that, in general, the blade airloads are not computed--rather, empirical values are used based on a predicted angle-of-attack distribution. Most simulations predict the structural dynamic response of the blades and use this together with an inflow velocity and wake induced velocity to define an instantaneous angle-of-attack for each airfoil section and then "look-up" the lift and pitching moment from tables of wind tunnel data obtained in steady state conditions. (Recall the question of definition of the unsteady angle-of-attack.) In some simulations, corrections are applied in an attempt to account for the unsteady effects. More recently oscillating airfoil wind tunnel data has been utilized in an attempt to include the unsteady effects. The rationalization for the use of these semi-empirical methods has been that they include some of the real fluid viscous effects and that the data can be obtained for each specific airfoil; but it is also recognized that there are deficiencies in such approaches.

The airfoil operating conditions for the two dimensional data (i.e., the stimulus which is the cause of the lift and moment response) can never match the conditions where it is to be applied within the rotor simulation. This is true for both the steady state and unsteady airfoil data. The unsteady airfoil data is generally obtained under the specific operating conditions of oscillating pitch or plunge and under the condition of simple harmonic motion (i.e., at a single frequency). Thus to be adequate it would seem that the data must be collected for all combinations of a, a, b, and reduced frequency. It should be noted that such data can only include the influence of the first two chordwise components of the airfoil stimulus (described previously). Furthermore there is serious question as to whether the airfoil lift and moment response (integrated measures of pressure response) to a general time variation of stimulus can be adequately synthesized from the component single frequency responses. For example, the sum of the individual component responses obtained from the data will not necessarily reproduce the instantaneous boundary layer conditions (i.e., state, thickness, and separation) which are actually present. The resulting lift and moment will be correspondingly influenced.

If the aerodynamic aspects of rotor aeroelastic simulations are to be improved so as to be consistent in accuracy with that which is possible in the structural dynamic aspect, adequate techniques for predicting the airloads (i.e., the lift, moment, and pressure response) will have to be developed. It is not useful to include the higher order effects in the structural representation, while ignoring them in the aerodynamic representation. Our future efforts should emphasize prediction of the airloads and deemphasize the use of empirical values.

Attempting to predict the unsteady pressure response of an airfoil may, at first, seem impractical or unrealistic but I recall that similar opinions were offered when it was first suggested that a detailed representation of the rotor blades and wake be formulated to predict the non-uniform flow. The capability to predict the pressure response is not as remote as it may seem because the essential elements, or at least their beginnings, are already available. For example, References 9 - 11 report procedures for predicting the steady state pressure response and, more recently, Crimi (Reference 12) has attacked the unsteady problem. These techniques generally iterate between the non-viscous potential solution for the airfoil pressures and a solution for the boundary layer response to these pressures. The boundary layer response is then used to modify the "effective" shape of the potential airfoil and/or the boundary conditions and the iterative cycle repeated until convergence is observed.

As described previously an important aspect of the airfoil flow which controls its pressure response is the attached near wake of shed type vorticity streaming from its trailing edge. Good representation of this portion of the wake will be required for prediction of the airfoil potential flow pressure response in the above described iterative scheme. The equally spaced concentrated shed vortex representations of this near part of the rotor wake which are generally used, will yield very poor results; this has been demonstrated in Reference 5. Subsequently, however, a technique was developed (Reference 13) which yields excellent results by effectively utilizing a continuous near shed vortex wake. Figures 5 and 6 (from Reference 13) illustrate the improvement attained. Each representation (i.e., discrete and continuous) was utilized to predict the lift and pitching moment as a function of reduced frequency for a two-dimensional airfoil oscillating in pitch and plunge. In these figures, the computed results are compared with the classical closed-form solution. The parameter, X, labelled as the number of shed vortices per cycle corresponds to the number of computational time increments per cycle of the airfoil motion. For the continuous representation, it is observed that the improvement is significant when X = 3 time increments per cycle and that the agreement is virtually exact when X=8 time increments per cycle. It should also be noted that the observed differences in correlation achieved with each wake model are the result of the differences in the wake representations but that the degree of overall correlation with the theory which was attained in each case is the result of both the wake and airfoil representations used. It is further observed that this combination of representations can accurately reproduce the theoretical unsteady lift and moment response to both pitching and plunging motions. This is indicative of the capability of these representations to simulate the airfoll pressure response to all four chardwise components of the stimulus. Note, that while airfoil pitch and plunge motions contribute directly only to the first two chordwise

components of the velocity distributions normal to the chord, the attached near shed wake will introduce relatively large contributions to all four components.

The preceding discussions of available representations which may be useful for implementing a method for predicting airfoil pressure responses was not intended to suggest that the described representations are necessarily the specific forms or procedure which should be pursued. They are presented merely as examples (not a complete survey of available methods) of what is possible with the present state-of-the-art and to thereby hopefully encourage future efforts to develop adequate means for predicting airfoil pressure responses.

# 3. SOLUTION TECHNIQUE

I now want to consider that aspect of rotor aeroelastic simulation which includes the numerical procedures and implementations utilized to obtain consistent solutions from the structural dynamic and aerodynamic aspects. There are two general classes of solution procedures in use. First there are the "forward integration" methods which are (in principle) capable of handling the unsteady flight conditions as well as the steady-state flight conditions (for which the rotor system responses are periodic). Transient calculation methods may be relatively inefficient for the steady-state condition because they must integrate over several rotor revolutions to obtain a periodic solution in the airloads, wake, and blade responses. The analysis of an unsteady flight conditions must of course first obtain the steady-state solution to provide the proper initial conditions. The second class of solution procedures (direct/iterative) is restricted to steady-state flight conditions and, conceptually, obtain direct solutions. In these procedures the system structural dynamic response is obtained by a direct numerical solution of the equations of motion for forcing functions based on the current approximation to the airloads. These system responses (blade motions) are then used in the aerodynamic representation to re-evaluate the airloads, and thus the forcing functions. This cycle is repeated until convergence, hopefully, is obtained. Thus this second class of solution procedures is iterative.

All rotor aeroelastic simulations (with which I am familiar) are similar in that they effectively have separate structural dynamic and aerodynamic representations which are coupled together by the solution procedure. That is, solutions are alternately obtained from each aspect of the simulation by using the results from one serve as input to the other. Furthermore, all apparently use the blade airloads as the basis for forcing function to the structural dynamic representation. This, I believe, is the primary source of the convergence problem which seems to periodically plague all rotor simulations.

The forcing function which stimulates the structural dynamic response of a system is a function of time and independent of the system dynamic response. The blade airloads, however, contain not only the aerodynamic forcing function but also the aerodynamic response forces which are functions of the structural response of the blades. These response forces are the aerodynamic mass, damping, and spring forces. Thus, as a function of the rotor operating conditions, they influence the natural vibration frequency and relative damping of each degree of freedom and thereby the system dynamic response characteristics. However, because these aerodynamic response forces are implicitly included within the aerodynamic representation and cannot be extracted, the total airload is generally used as the forcing function.

Using the total airload as the forcing function can conceptually compound the error. First, that which is being used as the stimulus (the forcing function) for the structural dynamic representation is in error, and secondly, the aerodynamic portion of the response controlling forces are completely missing from the structural dynamic representation. A partial compensation for these errors has been used in some simulations. This is accomplished by subtracting a quasi-steady approximation of the response forces from the airloads and including them in the equations of motion. This has been partially successful—it helps sometimes! Improving the accuracy of these approximations of the response forces should help improve convergence even further. It is obvious that if the correction were exact then the forcing function would be isolated and the first iteration would be the solution.

In some iterative types of solution, the solution process has been considered to possess what is termed a "closed-loop gain". It relates the change in the system response obtained from the current iteration to the corresponding change obtained from the prior iteration. This concept has led some investigators to introduce gain factors into the solution process in an attempt to control the convergence. This also helps-sometimes!

In the literature there exist standard procedures for assessing and improving the convergence properties of iterative computational techniques—they should be actively pursued. At the same time, it should also be kept in mind that the problem derives from the aerodynamic response forces being combined with the actual forcing function.

My next concern relative to solution techniques is closely related to the convergence problem--it is what I call (if you will pardon the expression) "solution pollution". By our solution techniques, do we necessarily get the solution to the

analytical representations used--or is there superimposed on the solution the response characteristics of the solution technique? This comment applies, I think, primarily to the iterative modes of solution and is evoked by the observed convergence problems. Intimately coupled with this question is the parallel question--how do the various solution techniques behave when the analytical representation of the system is lightly damped or even negatively damped? The forward integration can be expected to diverge, but then on occasion that happens when they are applied to systems which were supposedly stable! How will or should the iterative methods (not the system represented) react?

# 4. VALIDATION CONSIDERATIONS

I would next like to focus attention on the validation aspect of our efforts to develop rotor aeroelastic simulations. I believe that the lack of a significant improvement in the correlation of large-scale simulation results with measured results is in a large part due to inadequate validation efforts. Validation is the process of comparing the predicted results of the simulation with experimental results obtained from the system being simulated. The objective is to determine both the range and the degree of correlation between them and thereby the accuracy or usefulness of the simulation. However, in addition, a useful validation procedure must also provide information as to the source of the discrepancies. But, I believe, it is in this latter regard that most validation efforts are lacking. For adequate validation, the airloads and dynamic response must be resolved into meaningful components for the comparisons. This will not only allow more critical comparisons to be made by providing alternate and more detailed views of the results but may also provide additional information relative to the discrepancies.

# The Overall Simulation

Customarily comparisons are made on the basis of the time-history of the dependent variables such as blade airloads and bending moments at specific radial stations. These quantities (dependent variables) are essentially the objectives of the simulations. Thus, to obtain a subjective assessment of the degree of correlation achieved by the overall simulation, it is not unreasonable to view these time-histories relative to the measured results. However, the masking effect of the dominant low-frequency components in the total response can make it difficult to assess the accuracy with which the secondary components have been reproduced. That is, tasting the soup will not yield much information as to its ingredients--certainly not enough to allow it to be reproduced! Thus resolving the system responses into their components and viewing them may reveal much more diagnostic information and it will provide another view of the results. The simplest and most obvious (although not necessarily the most useful) component resolution would be into radial distribution of the harmonics. The "generalized airload" i.e., the spanwise integral of the product of the airload and the mode shape) for each blade degree of freedom at each harmonic is another possibility--many others are surely possible. Such component resolutions of the structural dynamic response and the aerodynamic response (or even combinations thereof) should be utilized in the validation efforts to enable critical and informative comparisons to be made.

# Component Representations

The above discussion was primarily concerned with validating the end or overall results of the simulations. However, closely related to this is the possibility of individually validating the component representations used within the overall rotor aero-elastic simulation. This is especially important in the large complex simulations we are dealing with. For example, if the errors in the airfoil stimulus due to the wake representation are relatively large, then an actual significant improvement in the airfoil representation may not be apparent in the end results. In fact it could degrade the results if originally the combination of errors (airfoil and wake) was compensating. I think that the validation of the component representations which are used in rotor simulations has been virtually non-existent.

Individual validation of these component representations can be accomplished by use of specially designed experiments and/or higher ordered analytical closed-form solutions. This would require that the representations be utilized in specially written simulations to treat the validating case. Care must be exercised to isolate, if possible, the various aspects to be simulated. The cause and effect relationship provides a useful division where the causal aspect can be precisely isolated as a known quantity. For example, two-dimensional oscillating airfoil test data could be used to validate airfoil representations—the input (cause) stimuli would be known. However, as previously discussed, the airfoil representation can be considered to be composed of a pressure response representation and an attached shed wake representation. It would not be possible to separate the effects of these aspects experimentally. Discrepancies between the predicted and measured results could be in either aspect. At this point the classical potential solution for an oscillating airfoil could be used to assess the adequacy of the attached wake representation as in Reference 13 and previously discussed (results presented in Figures 4 and 5).

# Data Accuracies and Simulation Inputs

Validation of rotor simulations depends not only on the procedure but also on the accuracy of the data used. One of the first steps taken in the effort to develop rotor aeroelastic simulations was the in-flight measurement of rotor blade dynamic responses and pressure differentials in the early 60's (References 14 and 15). These data are still used (as they should be) for various purposes. The point to be made (as was

demonstrated in Reference 16) is that the "generalized airloads", based on the measured data themselves, contain very large errors at frequencies above the third harmonic. Thus reliable comparisons between the predicted and measured results may not be possible at harmonics higher than the third. Noise calculations based on these data must also be of doubtful usefulness in the higher frequency range.

On the basis of the above observations and the advancements which have been made in data acquisition and handling techniques since these early tests, the following is recommended. Additional full-scale flight data should be obtained for, say, the first 10 harmonics of the blade dynamic response and pressure distributions. This data should be analyzed for at least three successive rotor revolutions to determine the degree to which the results are truly periodic. However, because of the many sources of random variations in the real-world situation, it must be recognized that the data can never be truly periodic. Thus, to provide meaningful data for validations (at all harmonics of concern), twisting techniques for establishing the statistical significance levels of data must be applied.

Closely related to the above considerations is that of the accuracy of the input parameters for the simulation. The tendency has been to include more details (effects) into the representation of the system. But how well can the magnitude of some of these parameters really be determined? If their influence on the result is considered significant then their values must also be significant. However, distributions (spanwise, normal, and chordwise) of many of the blade parameters are elusive and sometimes even difficult to control during manufacture. For example, it is often necessary to "slug" the blades with concentrated mass elements to match even the fundamental integrated measures of the mass distribution! Therefore similar questions relative to local variations in the blade stiffness parameters, shear centers, etc. Modern assembly methods no doubt provide "good" control--but "good" is relative and should be quantified.

# PRACTICAL USEFULNESS - APPLICATIONS

Careful consideration will, I believe, reveal that the usefulness of rotor aeroelastic simulations is actually rather limited. As presently formulated, they have the capability to solve what can be termed the "forward problem", i.e., given the specifications of the rotor system and its operating conditions, what will be its aerodynamic and structural dynamic responses? Certainly it is of interest to know how a proposed system will perform. However, if the system performance is not adequate, how should the system be modified or more fundamentally what is the system that will satisfy the performance specifications? These are the questions the designer faces and the present rotor simulations are of little help except in a trial and error fashion.

The design problem can be termed the "inverse problem", i.e., given the performance requirements (loads, response, noise, weight) or constraints, what is the system that will satisfy them? While there has been some effort expended on the inverse problem, it is very small relative to that devoted to the direct problem. Two examples, of inverse problem solutions with which I am familiar, are presented in References 17 and 18. The first of these seeks the rotor configuration for minimum power under the specified operating conditions. The second solves for the control system input requirements to eliminate specified components of the rotor vibratory shears transmitted to the fuselage.

The solution technique of Reference 18 is sufficiently general and novel to warrant a brief explanation here. The method involves only a minor conceptual modification (generalization) of an existing rotor simulation. The simulation used is of the "direct/iterative" solution type previously described, i.e., it obtains the periodic blade response by solving the equations of motion directly and iterating between the aerodynamic and structural dynamic representations. The coupled normal modes are used to represent the blade response and, because of the periodicity of the solution, these modal responses are represented by a fourier series. Thus the solution is found in terms of each harmonic of the response in each mode.

For the objectives of Reference 18, the blade root displacement equation of motion was added to the set of equations, the displacement in this degree of freedom was assumed zero, and the blade root shear treated as a dependent variable. The only other modification to the set of equations of motion was the addition of a constraint equation for each blade pitch control mode. The equations of motion for the pitch control modes were not included because the control characteristics were not considered in this study. However, the displacements in the control modes are included in the column matrix of variables. Thus a well-defined problem results for the complete system of equations having the same number of equations as unknowns. The solution procedure is exactly as in the original simulation, only the set of "equations of motion" has been slightly altered.

With this formulation it is now possible to specify the <u>value</u> (zero if desired) of any harmonic of the root shear and the solution will yield the control schedule required. Furthermore, via the constraint equations, it is possible to select and constrain any other system response variable as an alternate to the root shear. The value of the root shears will then be one of the results. It is even possible to operate it (the computer program) as a conventional rotor simulation by using the constraint equations to select the control displacements as the "variables" to be specified.

This technique can be generalized to do much more. The results are, of course, no better than the representations utilized within the simulation. It should be noted

that this approach did not require additional approximations to the representations of the original simulation. Furthermore the computational effort (running time) is virtually the same as for the rotor simulation upon which it was built. We are presently involved in an effort which will modify and use this technique.

#### SYNOPSIS

The following is a brief outline of what has been discussed.

 There has been significant progress made in the development of rotor aeroelestic computer simulations.

However the progress has been primarily in the expansion of the scope of their predictive capability.

Over the last 10 - 12 years, the improvement in the correlation of the predicted and measured results has not been significant.

- The available technology of structural dynamic representation is not presently limiting the capability of rotor aeroelastic simulation.
- The available technology of aerodynamic representation is the least developed and is limiting the capability of rotor aeroelastic simulation.

Progress has been made in developing rotor wake representations but they are still far from adequate.

The blade airloads are not being predicted! The blade airfoil section representation is essentially empirical.

Accuracy of the aerodynamics comparable with that of the structural dynamics will require that blade section aerodynamics be predicted. It will be possible.

- Convergence problems of the simulations derive from the manner in which the blade airloads are used as the forcing function.
- Validation efforts relative to simulation development have been less than adequate.

Validations should be more detailed, critical, and informative.

Validation of the component representations within the simulation should be made.

 The actual usefulness of rotor aeroelastic simulations in their present format is somewhat limited.

Present formulations predict what the performance of a  $\underline{\text{specified}}$  rotor will be.

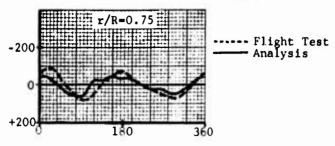
A more useful formulation, for the designer, is one which will solve the inverse problem. That is, given rotor performance requirements (i.e., constraints), what is the rotor system that will satisfy them.

The topics covered in this discussion are essentially an outline of those areas where I believe efforts should be concentrated to obtain significant improvements in rotor aeroelastic simulation.

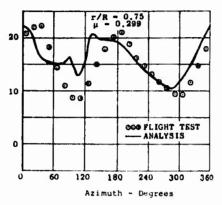
# 7. REFERENCES

- Willmer, M.A.P.: "The Loading of Helicopter Rotor Blades in Forward Flight" ARC R&M 3318, 1963. Replaces R.A.E. NAVAL No. 2, April 1959.
- 2. Miller, R.H.: "On the Computation of Airloads Acting on Rotor Blades in Forward Flight" Journal of the American Helicopter Society, April, 1962.
- 3. Piziali, R.A.; and DuWaldt, F.A.: "A Method for Computing Rotary Wing Airload Distributions in Forward Flight" TCREC TR 62-44. November 1962.
- 4. Piziali, R.A.: "A Method for Predicting the Aerodynamic Loads and Dynamic Response of Rotor Blades" USAAVLABS TR-65-74, January 1966. Also, Journal of Sound and Vibration, Vol. 4, No. 3, (pp. 445-489), 1966.
- Crimi, P.: "Theoretical Prediction of the Flow in the Wake of a Helicopter Rotor" CAL Report No. BB-1994-S-1 and BB-1994-S-2, September 1965.
- Scully, M.P.: "A Method of Computing Helicopter Vortex Wake Distortion" MIT ASRL TR 138-1, June 1967.

- Landgrebe, A.J.: "An Analytical Method for Predicting Rotor Wake Geometry" AIAA/AHS Georgia Tech VTOL Aircraft Meeting, September 1968.
- Sadler, S.G.: "Main Rotor Free-Wake Geometry Effects on Blade Air Loads and Response for Helicopters in Steady Maneuvers" NASA CR-2110, September 1972.
- 9. Spence, D.A.: "Prediction of the Characteristics of Two-Dimensional Airfoils" JAS, V-21, N-9, September 1954.
- Preston, J.H.: "The Calculation of Lift Taking Account of the Boundary Layer" ARC R&M 2725, November 1949.
- Maskew, B: "A Surface Vorticity Method for Calculating the Pressure Distribution over Aerofoils of Arbitrary Thickness and Camber in Two-Dimensional Potential Flow" Loughborough University of Technology, TT 6907, April 1969.
- 12. Crimi, P., and Reeves, B.L.: "A Method for Analyzing Dynamic Stall of Helicopter Rotor Blades" NASA CR-2009, May 1972.
- 13. Daughaday, H. and Piziali, R.A.: "An Improved Computational Model for Predicting the Unsteady Aerodynamic Loads of Rotor Blades" Journal of the American Helicopter Society, Vol. 11, No. 4, October 1966.
- 14. Burpo, F.: "Measurement of Dynamic Airloads on a Full-Scale Semi-Rigid Rotor" TCREC TR 62-42, December 1962.
- 15. Shieman, J.: "A Tabulation of Helicopter Rotor-Blade Differential Pressures, Stresses, and Motions as Measured in Flight" NASA TM X-952, March 1964.
- 16. DuWaldt, F.A., and Statler, I.C.: "Derivation of Rotor Blade Generalized Airloads from Measured Flapwise Bending Moment and Measured Pressure Distributions" USAAVLABS TR 66-13, March 1966.
- 17. Tung, C. and Brady, W.G.: "Study of Rotor Configuration for Minimum Power and Rotor-Wake Pressure Distributions" CAL Report BB-2293-S-2, September 1967.
- Daughaday, H.: "Suppression of Transmitted Harmonic Rotor Loads by Blade Pitch Control" USAAVLABS TR 67-14, November 1967.



a. Earlier Results (ref. 4).



b. Current Results.

Figure 1. Examples of Early and Current Correlations.

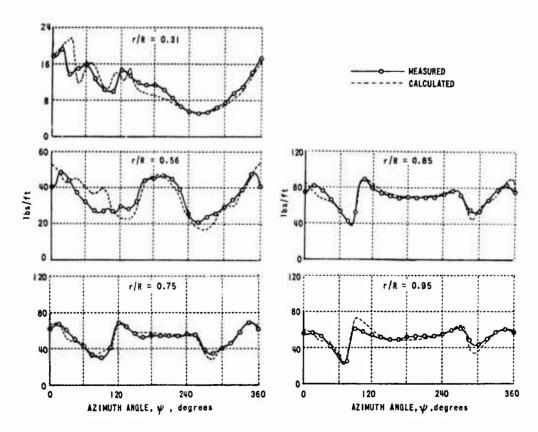


Figure 2. Example of Early Correlation (ref. 3)

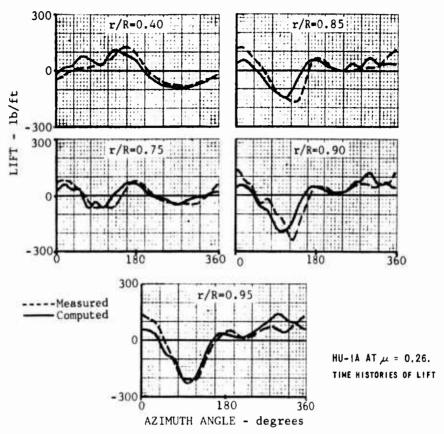


Figure 3. Example of Early Correlation (ref. 4).

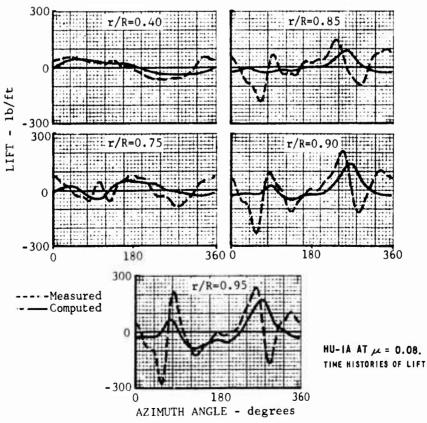


Figure 4. Example of Early Correlation - Poor (ref. 4).

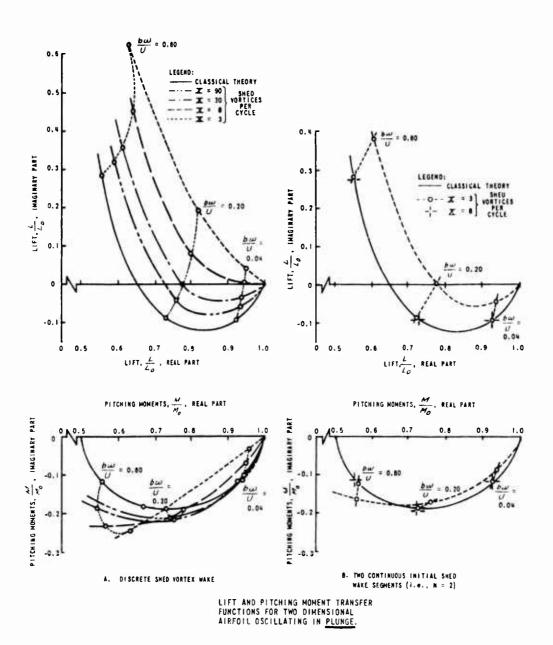


Figure 5. Discrete and Continuous Wake Correlation with Theory (ref. 13).

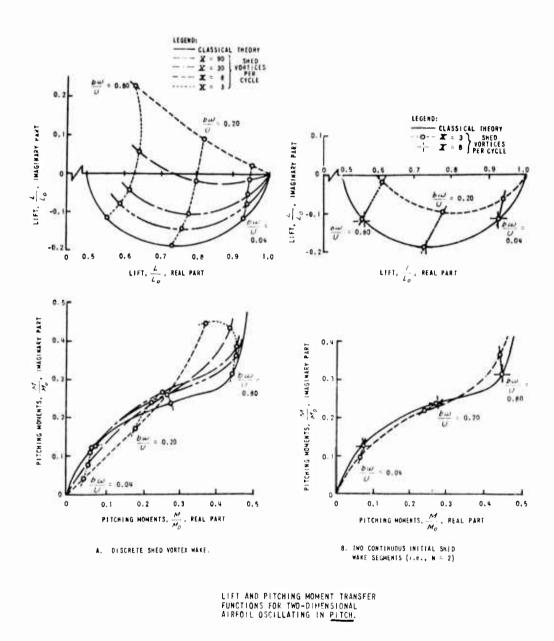


Figure 6. Discrete and Continuous Wake Correlation with Theory (ref. 13).

# Discussion of Paper 1 "Rotary Wing Design Technology" presented by A.Z.Lemnios

W.Z.Stepniewski: What was your tip speed?

A.Z. Lemnios: In the 101 rotor? I believe the current tip speed is 661 fps.

# Discussion of Paper 2 "Current Loads Technology for Helicopter Rotors" presented by R.Gabel

**R.L.Bennett:** In your solution of the equations of the lumped masses did you use all twenty lumped masses? Differential equations associated with all of them?

R.Gabel: Yes, but it is not a direct differential equation, it's an iteration around the azimuth.

J.L.McCloud: Mr Gabel, could you comment on the scatter differences between the various radial stations and the various figures you have shown. It does not seem to be a consistent pattern in the scatter, I am referring to the bending moments.

**R.Gabel:** I think that really comes with the magnitude of the flight test sample. The 347 has the smallest scatter and also we have the smallest number of flight data points. The Chinook has a vast number of flight data points over many years of flight testing and the extremes of scatter tend to be the largest although they tend toward a mean which is pretty consistent; the extremes just get larger with more data.

J.L.McCloud: Then this is not the scatter of one particular run, it is the scatter of many, many flights?

**R.Gabel**: It is really referring to specific runs. However, when you get a long run and read many, many cycles of data, the same effect happens.

**W.P.Jones:** In your very last sentence—or last sentence but one—you mentioned that more concentration were to be devoted to study transient effects. What did you have in mind here, was it just the dynamic stall or the blade tip vortex interaction or gust effects?

R.Gabel: It is some of all these things. When the aircraft maneuvers at the edges of its envelope it causes loads which go deeply into stall—or can go deeply into stall—and can encounter vortex strikes which generally are most effective in increasing bending moments. I think there are really almost two things: the stall increases the torsional loads and the pitch link and control load in maneuvers, whereas the vortex strikes tend to enlarge the bending moments on the rotor and both of those effects I think need to be addressed in future transient load work.

W.J.McCroskey: In another part of your presentation you had a graph of pitch link loads versus air speed with predictions and data, predictions based on what you said were quasisteady aerodynamic characteristics for the airfoil in the program and unsteady characteristics. Now my question is: The curve of what you call quasisteady airfoil characteristics, did those include stall, static stall characteristics, because it looked like a curve which is just the continuation of unstalled pitching moments and you showed a large difference between the so called quasisteady prediction and the unsteady prediction. I wonder how much of that is due, or is all of it due to unsteady stall effects or is some of it due to stall and some due to unsteady stall?

**R.Gabel:** The quasistatic stall is a straight line with, I believe, a  $12^{\circ}$  angle cut-off, that we used when those were done, the unsteady is the representation of the hysteresis loop in somewhat empirical fashion based on test data, a  $\gamma$ -function as we call it.

W.J.McCroskey: So static stall cm's were in the curve that you called quasistatic?

R.Gabel: Yes with a cut-off at twelve degree, yes.

# REVIEW 1 by A.J.Sobey Reply by A.Z.Lemnios

With regard to the questions that Mr Sobey raised, in our normal mode approach I specifically avoided that use of the word normal mode, because they are in fact not normal modes, they are uncoupled modes; I differentiate between the two and I refer you to the textbook by Mr Scanlan and Rosenbaum for detailed explanation of the two, in that the uncoupled modes are orthogonal onto themselves in the flapwise, the edgewise and chordwise direction and in the torsional direction, however they are not orthogonal one to the other, and so incorporating these modes we make use of the orthogonality conditions when they are applicable. However, in the governing equations of motion we do keep the coupled terms in there, so they do interact with each other. The modes themselves are derived on the basis of uncoupled analysis, independently, and I think you are correct to question the validity of whether we should use linearized modal analysis here or not. However, I can only state the fact, that they seem to give us good approximations and good responses.

With regard to the nonlinear terms and the retention of the various terms in the equations of motion, we do in fact retain large angles, large motions when they are derived through our techniques which is primarily nothing more than virtual work.

In order to check on the stability of the system itself, the terms that we talked about and what I called the A-matrix, the linear part of the solution, include linearized terms in accelerations, displacements, velocities; however they do include approximations to nonlinear aerodynamics and compressibility corrections and approximations to stall effects, as well, aerodynamically; so we do have some nonlinear terms in the equations for stability characteristics

To your second point: that was the integration of load distributions and the technique applied here is the "forward notch" approach whereby a method is to assume an initial value and see if we do have convergent solutions and in this situation, on a numerical approach, you may come up with an instability, and the natural question that arises in that circumstance is: Is the system itself unstable or do you have a numerical instability? And that one has been a problem to all of us. In some instances we do have system instability however in others we have numerical instabilities and not physical ones.

We have in the Berman technique first a periodic response by converting the initial value matrices to boundary condition matrices by forcing the vector at the  $360^{\circ}$  azimuth to be identical to that at the  $0^{\circ}$  azimuth and thereby, whenever we do get a solution at anyone particular instance it is periodic within itself. That is not to say that it has converged, however we do test for convergence between one iteration to the next iteration and force periodicity in this instance.

Your question regarding: do we have to assume periodic solutions, is a very valuable one and your reference to the split tip path plane is an excellent example. I think as a first response — as a first order approximation in the first design point assuming periodic solutions is a good analysis and a good start. It is not necessarily a final one however, and I agree with you that if we were to try to fully understand and fully predict the responses and stability characteristics of the rotor system we should, in fact, take into account a fully coupled modal analysis of the blade, the impedance matching at the hub, the control system and perhaps even some structural dynamic terms from the fuselage itself.

However, I believe that—and this is conjecture on my part and I can't prove it but I have a strong feeling that if we were to attempt this, that by the time we reached a numerical solution we would have long flown the machine. So that it becomes a trade-off as how much of a shine do we want to put on the apple? Do we want in fact to put such a high luster and such a refinement on an analysis that we lose sight of our objective which is to make aircraft and make them fly. Analysis is great unto itself and I am interested in and all in favor of doing analytical work, since it is my bread and butter, however I am equally interested in building hardware and making it fly, too—and we have to have a trade-off here as to where do we have to stop one and to begin the other.

With regard to the Liapunow stability criteria, I have looked at that and I am sorry to admit I am not much of an expert on these nonlinear stability techniques and we have not really examined it to the extent that we should, in trying to apply such techniques to rotor system stability.

### Reply by R.Gabel

Regarding some of the points Mr Sobey made first on our analysis it was not clear, I guess, that we use an associated matrix technique which was started some twenty years ago by Mr Targoff who was then with Glenn Martin. The analysis relates each bay elastically and masswise with a set of boundary conditions and multiplies itself into the root and then repeats this for each azimuth. We don't use modal on this program; but I don't say that there is something wrong with modal. I think in fact it is probably better, because it reduces the amount of mathematics needed and we just happen to get started down the path of the matrix type.

As regards the modal content in the flight test data you can approach that either by having three or four modes in the modal approach or in the concentrated mass program like this one that has some 25 masses, that's equivalent to about 24 modes, that's overdoing it from that point of stand.

We have the same problem with the iteration as Dr Lemnios mentioned and the same questions about stability when we get to the end and we use the periodic solution in the dynamic side, that is we do the aerodynamics without it and then harmonically analyze the air loads, apply them harmonically to the blades and get the response harmonically, add them back together again, to get the total response and then go back to iterate the aerodynamics, this is a long process that computers only make possible.

With regard to the full coupling in the C-70 program, that is fully coupled flap, pitch and lag, I did not make it clear, that the application shown here, was for a highly twisted tilt-rotor airplane that has about 35° built-in twist. And it is in these applications we really think a full coupling is needed. However there are other applications in helicopter small twist blades, where it is nice to have a full coupling capability to explore travelling, neutral axis, or shear centers that don't necessarily go in a straight line from the pitch axis, which we have not been able to do with the other types.

The projection that the 5% damaging bads is a key one and it is rather crude what we are doing, and — as it was mentioned earlier——that's the area of greatest analytical need

There are other roads of backing up the guess so to speak which we are going, and we have considerable resources going into model testing, where with the new data we are going to derive new airfoils and new concepts involved which are not like previous rotors. We do extensive dynamic modelling and measuring loads on these dynamic models as another cross check on the capability of the analysis. But that is indeed a weak area.

# REVIEW 2 by R Piziali Reply by A Z Lemnios

Just a couple of points with regard to Mr Piziah's comments about improvement in correlation. Prediction techniques historically have not improved significantly in the past few years. I think that there has been a significant improvement from the late fifties to approximately the mid to late sixties with a signif-ant jump there with the introduction of the computer.

I tend to agree with him that since that quantum jump we had had some improvements that were not as significant as that initial jump and to back up my feeling on that. I refer you to the work by Bellinger at UARL, which I think is rather significant, in which he stated the higher modal introduction into the analysis to give you some improvement in performance but primarily from the low bending moment stand point, so that as far as performance and rotor forces are concerned the fundamental modes are the more important ones, and those are limited to the first one or two essentially.

With regards to your comments on airloads not predicted directly that is you calculate angles of attack, and you do a table look up; this again gets back to the age-old question of synthesis versus analysis. We can always analyze something but we find it very difficult to synthetize something; I agree with you completely that ideally what we would like to be able to do is to specify rotor requirement and let the computer define the airload distributions, and then have some sort of a predictive technique in there, which will find the airfoil cross-sections for us and its pressure distributions both steady and unsteady. Now, if you can imagine that in such a program you are a visionary. A first step at least in this direction—and one which we have attempted to take is the approach we are using in our Controllable Force Rotor, in that by independently controlling the outboard and inboard ends of a soft blade, we are attempting to redistribute the airloads, and are attempting in fact to somehow optimize. On the question of optimization you mentioned power; however power is only one parameter doing our CTR study we initially have looked into power as a sole parameter and found that it leads down the primrose path. It turns out that in addition to power you must also monitor blade stall boundary, you must also monitor blade dynamic response in order to get a handle on blade life and also to get a handle on vibratory loads in the hub, which in essence control the structural response of the fuselage. So it becomes a multiparametric study and not just single parametric study, you are not optimizing power only, you are optimizing three of four independent parameters and when you do have a complex surface response such as this, it becomes a n-dimensional figure and you can no longer mentally capture it and it becomes extremely difficult to analyze. This is where the iteration procedures and response surface techniques come into play.

# Reply by R.Gabel

A very brief response: I agree with Mr Piziah about the correlation, I think the correlation has come a long way, and I would protest there has been a lack of progress in correlation over the years.

With regards to improving the wake analysis, I think you are right that a lot can be done about that and what we are using is crude. Things like the torsion effect, that you mentioned, are quite important. In fact there are aspects of other problems such as torsional divergence possibilities, that are coming into force. Rotors get more heavily loaded and go to higher speeds, but also these analytical programs need to be tested against. As regards validation of components, I agree with the comment there that the airload distributions and such things do need to be looked at.

The dynamics part is very easy to check, that is you can take the rotor blade and do ground shake test on it and determine the modes and the modal locations and check that portion of the calculation, which is purely dynamic.

There are some other things like the shear center and such locations, they are all difficult to find, difficult to measure even statically; although you get some results on them by doing a shake test allowing flap and torsion to couple, so you can check that part of the dynamics. And also these analyses can do some of these exploratory things like second harmonic control. We have used the C-60, it has the capability for second order harmonic control and we can make use of it.

J.J.Cornish: I want to comment on how strongly I feel about the comments you made regarding the aerodynamics being the problem of the problem. Until the aerodynamics of the system is defined a bit clearer, I don't think we will approach solving the problem any further, and I'd like to suggest that perhaps we can break the aerodynamic problem into two regimes in studying the cobblestone road that is mentioned. Both of them are due to local changes in angle of attack as the blade sweeps around. But, perhaps there are two different kinds of things. One is a variable in the downwash in a relatively continuous wake of the flow field and the other one is the existence of the discrete vortices which lie in the wake. These are similar to the laminar and turbulent boundary layer air flow whereas in the one part there is only a vorticity present and in the other there are vortices present. I think that the analysis of the load when the aircraft is fluctuating along the linear portion of the lift curve present the former or laminar type and as the flow begins to separate or come over the nonlinear portion of the lift curve perhaps then you begin to shed vortices. I'd like to make another comment that it is not really too visionary to suspect that we can have the inverse problem. We've already done some of this wherein we could prescribe the pressure distribution shape that you wish the aircraft to have and then we plug it into the machine and then out comes the airfoil shape. We did some of this at Lockheed and it is being continued at Bell and it can also be made time dependent. I think the real problem is that when we stop doing this and get into the nonlinear aspects. We don't know how to handle the nonlinear part of the lift curve because we do not know when the separation occurs. I came as a matter of fact, hoping to hear the answer to the question that I'm going to have to leave with you, and I think it's a question we have to answer before we can go any further with the aerodynamic considerations, and that is the simple question f how much vorticity does it take to make a vortex?

**P.J.Arcidiacono:** I would also like to question whether or not the dynamics is well in hand. It seems to me that you can very rapidly set up a system of equations of motion representing springs and masses and so on. But one of the real problems is knowing the values of those springs beforehand and that is really a crux of the designers problem, for example modelling the control system swashplate integrated coupling. We have a very complex situation here. That could be a subject of research all by itself.

**R.G.Loewy:** I am very glad that Mr Arcidiacono said what he just did, because I think there is among the dynamic-ists the feeling of general euphoria with respect to structural dynamics. It is not warranted. I just would like to point out that, most of the real major rotor problems that come up from time to time and have come up recently, have been in this area.

J.L.McCloud: I will add to this same comment: I was wondering why I did not see a correlation between the bending loads realized and the aerodynamic loads which have been measured. I gather the problem is not that simple.

I also would like to comment that we have had some success with high harmonic control, but the problem is, can we considerably linearize and then ask the question in what direction should one go to try to improve at least the aerodynamic problem? If any one has some ideas on this improvement...

# CALCUL DES CHARGES SUR ROTOR D'HELICOPTERE

#### PREDICTION OF HELICOPTER ROTOR LOADS

par J. GALLOT Chef du Service Aérodynamique

AEROSPATIALE (France)

### SOMMAIRE :

Le dessin correct d'un rotor suppose une connaissance relativement précise des charges alternées auxquelles la pale et le moyeu seront soumises. Le problème de l'évaluation, dès le stade dessin, des contraintes peut conduire à des méthodes très sophistiquées, compte tenu de la complexité de l'environnement dans lequel fonctionne la pale. Néanmoins, des méthodes simplifiées peuvent donner des résultats suffisamment précis pour permettre un dimensionnement correct des principaux éléments du rotor. La méthode exposée ici suppose une aérodynamique très simple et indépendante des déformations élastiques de la pale. Le degré de simplification retenu parait justifié par la corrélation obtenue avec les charges aérodynamiques mesurées sur le rotor maquette Modane et les contraintes relevées sur ce même rotor et sur un rotor échelle grandeur.

#### SUMMARY :

The correct design of a rotor requires quite a precise knowledge of the alternating loads to which blade and hub are submited. The problem of the stress evaluation, from the early design stage, may lead very sophisticated methods, because the blade is operating in a very complex environment. Nevertheless simplified methods may give sufficiently precise results to set up correctly the dimensions of the main elements of the rotor. The method described here supposes simple aerodynamics, independent of blade elastic deformations. The degree of simplification achieved in this theoritical method seems to be justified by the correlation obtained with experimental airloads measured on a model rotor at the Modane Wind Tunnel, and stresses recorded on the same rotor or a full-scale semi-articulated rotor.

### HOTATIONS :

$$P_{M} = \int \frac{\text{Apc dx}}{\text{corde p0}}$$
 charge locale sur la pale

Λ p = pression différentielle locale intrados - extrados

c = corde

po = pression statique dans la veine

Ao, Ai, ... An = coefficients en cos n $\Psi$  de l'analyse harmonique de  $P_{\mathbf{m}}$ 

B1, B2, ... Bm = coefficients en sin n $\Psi$  de l'analyse harmonique de  $P_{M}$ 

Ψ = azimut de la pale (origine pale arrière dans le lit du vent)

A = paramètre d'avancement

de = inclinaison de l'arbre rotor par rapport au vent

0,035 = Pas général à 0,75 R

R = Rayon rotor

r = position en envergure d'un poste de mesure

# 1 - INTRODUCTION :

1 1

La prédiction des charges et contraintes sur pales d'un rotor d'hélicoptère est un problème essentiel au stade bureau d'étude puisqu'elle doit permettre d'orienter le choix au point de vue fréquence propre de pale et de donner les éléments nécessaires au dimensionnement correct des parties critiques du moyeu et des pales. Théoriquement une méthode bien au point devrait conduire aussi à une estimation précise des efforts alternés à la tête rotor et une fois connue la fonction de transfert du fuselage, l'évaluation du niveau vibratoire de l'appareil complet, deviendrait possible au stade du dessin. Malheureusement les outils développés aujourd'hui par différents organismes de recherche ou constructeurs donnent des résultats encore partiels à notre connaissance, dans une zône réduite de validité en ce qui concerne les grands paramètres d'avancement, la compressibilité en pale avançante ou le décrochage en pale reculante.

Par ailleurs les méthodes très sophistiquées qui ont été développées depuis de nombreuses années, n'ont pu voir le jour que grâce à l'apparition d'ordinateurs modernes, rapides et à grande capacité de calcul. L'utilisation de telles machines s'est traduite par une lourdeur d'utilisation croissante et un éloignement de la réalisé physique.

Le coût de développement et d'utilisation de telles méthodes ne nous a pas paru être à l'échelle des résultats obtenus de cette façon.

Pour ces raisons nous avons cherché à développer un outil de travail simple, pour ne dire simpliste, destiné à nous donner, les éléments nécessaires au dimensionnement des différentes parties d'un moyeu ou d'une pale.

# 2 - COMMENTAIRES SUR LA SOPHISTICATION DU CALCUL DES ROTORS :

Les méthodes de calcul de rotor doivent prendre en compte essentiellement trois catégories de problèmes qui s'imbriquent plus ou moins suivant le type de la méthode.

- 1) Représentation correcte de la dynamique de la pale
- 2) Définition du champ spatio-temporel des vitesses induites par le rotor lui-même.
- 3) Calcul des charges aérodynamiques sur la pale à partir de la connaissance de la répartition géométrique et de l'évolution dans le temps de l'incidence, du dérapage et du Mach des écoulements locaux sur la pale.

La pale est généralement représentée par ses modes propres dans le vide calculés par des programmes spécifiques rendant compte des différents couplages possibles entre battement, trainée ou torsion. Il est à noter que les données de base : masse, rigidité, centre de gravité, centre de torsion sont parfois imprécis notamment dans le cas des pales fabriquées avec des matériaux composites (roving, fibres de verre ou carbone etc...) Par ailleurs la validité des fréquences propres, masses généralisées, déformées est difficilement contrôlable. L'analyse d'essais sur un banc rotor ne donne es effet que des renseignements partiels sur la dynamique d'une pale en rotation.

La représentation élastique de la pale doit donc être considérée comme problématique au point de vue précision.

La représentation du champ des vitesses induites par le rotor a conduit au développement de nombreuses méthodes se différenciant par leur complexité plus ou moins grande. Certaines utilisent un sillage très simplifié (anneaux, lanière rectiligne, cylindres tourbillonnaires etc...). D'autres font intervenir un sillage de forme imposée hélicoïdale ou déformée empiriquement. Les plus évoluées s'attaquent à la mise en équilibre du sillage lui-même (voir références 1,2,3,4 par exemple). Une approche un peu différente utilisant la théorie du potentiel d'accélération donne aussi des résultats intéressants (référence 5). La validité des résultats obtenus ainsi ne doit pas faire oublier cependant que des recherches sont en cours sur les tourbillons eux-mêmes (stabilité, structure, intéraction pale-tourbillon).

Les charges aérodynamiques en finale sont calculées en utilisant une polaire bidimensionnelle stationnaire ou instationnaire synthétisée à partir de résultats d'essais harmoniques sur profils oscillants. L'influence de l'attaque oblique est rendue de manière empirique par augmentation des CZ MAX. et introduction d'une trainée radiale. Mais le traitement rigoureux des écoulements tridimensionnels a commencé seulement récemment en liaison avec des expérimentations spécifiques (référence 6), cette lacune étant très grave au voisinage du décrochage (extrémité de pale, cercle d'inversion) et dans le domaine compressible (extrémité de la pale avançante). Ces quelques remarques ont pour but de montrer non seulement qu'un travail important de recherche est encore nécessaire pour maîtriser la connaissance du fonctionnement aéroélastique du rotor, mais aussi que l'ingénieur au niveau dessin, doit s'appuyer sur des méthodes plus simples, lui donnant les informations qu'il demande, avec une précision suffisante.

#### 3 - DESCRIPTION DE LA METHODE DE CALCUL :

Le programme de calcul des contraintes est constitué de deux blocs relativement indépendants permettant de dissocier le calcul des charges aérodynamiques et la réponse dynamique de la pale (voir Rigure nº 1). Les données correspondant à un cas de vol hélicoptère sont obtenues à partir d'un programme de qualités de vol indépendant déterminant l'équilibre de l'appareil à partir des caractéristiques aérodynamiques du fuselage et d'une formulation algébrique du rotor principal. Ce programme définit notamment la position des commandes de vol et l'incidence du mât rotor, nécessaires pour le calcul relatif au rotor considéré ensuite comme isolé.

Le principe de base de la méthode est de supposer que le calcul des efforts aérodynamiques peut se faire simplement, en admettant que seul le premier mode de hattement (articulé ou rigide) intervient pour la détermination des vitesses engendrées par les mouvements de la pale. Il est clair que cette hypothèse ne pourra conduire à des résultats cohérents que pour un rotor bien adapté au point de vue fréquences propres, but recherché évidemment au stade avant projet. Par ailleurs la pale doit être suffissamment rigide en torsion (fréquence propre de torsion élevée, grande rigidité de commande), pour que les mouvements parasites en pas de la pale, ne deviennent pas trop importants par rapport à la commande cyclique et aux variations d'incidence induites par le fonctionnement du rotor. Il ne faut pas espèrer obtenir ainsi des résultats cohérents lorsque le rotor est franchement décroché, mais de toute façon des méthodes plus complexes ne semblent pas donner des résultats absolument fiables dans cette sône de fonctionnement du rotor. Cette hypothèse étant admise, avec les limitations qu'elle apporte, les charges aérodynamiques sont calculées par la méthode désormais classique utilisée dans la référence 8 :

- Calcul de l'incidence et du Mach pour chaque section de pale
- Détermination de Cz et Cx à partir d'une polaire bidimensionnelle de prof!1
- Calcul du moment des forces aérodynamiques par rapport à l'articulation de battement (réelle ou fictive)
- Résolution de l'équation de battement pas à pas
- Stabilisation du battement sur plusieurs tours pour atteindre un fonctionnement stabilisé correspondant à un cas de vol donné.

Le coefficient de pertes en bout de pale utilisé dans la référence 8 a été remplacé par une diminution progressive de la charge en extrémité, un peu semblable à celle utilisée dans les théories d'aile portante L'hypothèse initiale est en effet trop grossière lorsqu'on s'intéresse à autre chose qu'aux performances globales du rotor.

#### 4 - COMPARAISON DES CHARGES AERODYNAMIQUES CALCULEES ET EXPERIMENTALES :

Cette comparaison a été faite sur un rotor maquette de 4 mètres de diamètre, tripale, essayé dans la grande Soufflerie S1 de Modane en 1970 (voir figure nº 2). De nombreux résultats intéressants ont été obtenus avec ce moyen d'essai aussi bien au point de vue performances globales, visualisations d'écoulement sur rotor, qu'analyse fine des phénomènes locaux sur la pale. Quelques résultats ont été présentés antérieurement dans les références 9 et10. Le rotor était essayé dans des conditions réelles de vitesse d'avancement et de Mach en bout de pale. Les pales étaient équipées de capteurs de pression différentielle répartis selon 4 cordes situées à 0,52R, 0,73R, 0,855 R, et 0,95 R.

Le premier cas envisagé correspond au fonctionnement en croisière économique d'un rotor d'hélicoptère actuel, c'est à dire loin de l'apparition des phénomènes de décrochage et de compressibilité. La comparaison a été faite jusqu'au troisième harmonique des charges aérodynamiques, les harmoniques de rang supérieur à trois étant négligeables aussi bien expérimentalement que par le calcul (figure n° 3).

Le deuxième cas présenté correspond au fonctionnement d'un rotor à la VNE en piqué, juste avant l'apprirition des phénomènes de compressibilité en pale avançante. La comparaison est arrêtée dans ce cas à l'harmonique de rang quatre pour les mêmes raisons que précédemment (figure n° 4).

Dans les deux cas, compte tenu des hypothèses simplificatrices retenues, les écarts calcul-expérience sont suffisamment faibles pour permettre une évaluation satisfaisante des contraintes pales dans une optique dimensionnement pale et moyeu. La corrélation dans les cas de vol décroché est moins bonne, mais est sans doute améliorée par l'introduction des phénomènes instationnaires autour du décrochage d'après nos premières investigations.

# 5 - EXTENSION DE LA METHODE :

Devant les résultats acquis, déjà satisfaisants, la méthode a été légèrement modifiée pour tester l'influence de certains hypothèses simplificatrices. Tout d'abord pour améliorer la définition des charges en extrémité de pale, nous avons testé un modèle simplifié de sillage qui peut être décrit de la manière suivante (Figure n° 5):

- La circulation est supposée constante en envergure mais variable avec l'azimut de la pale
- De ce fait le sillage est constitué théoriquement d'un tourbillon d'extrémité de pale (A), d'un tourbillon de pied de pale (B), ainsi que d'une nappe radiale pseudo hélicoïdale (C)
- Le tourbillon (A) est décrit par une lanière se transformant très vite en une surface tourbillonnaire, pour diminuer le temps d'intégration (A)
- Le tourbillon (B) est décrit aussi par une surface tourbillonnaire (solénoîde à spires jointives).
- Quant à la nappe tourbillonnaire radiale associée au fonctionnement instationnaire de la pale, elle est remplacée par un volume continu de tourbillon.

Les résultats ont été comparés avec ceux obtenus par une méthode plus évoluée de calcul des vitesses induites (réf. nº 11). La planche 6 montre que l'on obtient ainsi une bonne approximation de ce problème. La corrélation sur les charges a été regardée, mais les résultats ne semblent pas être beaucoup plus précis (figure nº 7) que ceux obtenus par la méthode initiale. Néanmoins cette idée doit être explorée plus à fond, car elle permet notamment d'introduire le déorochage instationnaire des profils en gardant un sohéma global relativement cohérent. Nous testons simultanément l'influence des eff ls instationnaires suivant la méthode précomisée dans la référence (7), les Cs, Cv, étant recalculés à partir d'un tableau de données synthétisées en fonction de la valeur de l'incidence instantanée, de sa vitesse de variation et de sa dérivée seconde.

Nous espérons ainsi en testant séparément l'influence des différents phénomènes physiques et les hypothèses adoptées pour les représenter (pertes en extrémité de pale, champ des vitesses induites, attaque oblique, effets instationnaires) arriver à un modèle aérodynamique très simplifié, permettant un calcul suffisamment précis des charges sur pale d'hélicoptère.

# 6 - REPONSE DYNAMIQUE DE LA PALE :

Les propriétés élastiques des pales sont prises en compte en utilisant une représentation modale de la pale limitée en importance suivant les cas (3 modes de battement et 2 modes de trainée en général). Les caractéristiques modales utilisées sont issues d'un calcul de mode propre découplé compte tenu des simplications envisagées par ailleurs. La méthode de la référence (12) donne des résultats suffisamment précis en minimisant le temps de calcul.

La réponse de la pale est obtenue en calculant la contribution de chacun des modes propres excités par les différentes harmoniques des charges aérodynamiques. Afin de tenir compte approximativement des efforts aérodynamiques induits par les mouvements de flexion verticale de la pale, un terme d'amortissement aérodynamique linéaire évalué dans le cas du vol stationnaire est introduit pour les modes de battement autres que le premier mode rigide.

Les contraintes sont ensuite calculées par superposition de la contribution en moment des différents modes et ceci pour chaque harmonique. Il est à noter que la précision du résultat dépend beaucoup de la description fine de la pale au point de vue massique et élastique, et que les zônes de la pale rapidement évolutives telles que attache, manchon etc..., généralement critiques au point de vue dimensionnement, doivant être définies avec précision.

Le découpage retenu dans cette méthode, entre l'aérodynamique et l'élasticité permet très rapidement de chiffrer les répercussions d'une modification quelconque de la pale, à partir du moment où les données aérodynamiques sont stockées en mémoire.

# 7 - ANALYSE DES RESULTATS AU POINT DE VUE CONTRAINTES :

La méthode a été utilisée tout d'abord sur le rotor maquette Modane qui est articulé à la fois en battement et en trainée. Les pales de ce rotor ne sont pas dynamiquement semblables aux pales d'un rotor grandeur (nombre de Lock plus faible). Les déformations de la pale ont donc une influence très faible sur les efforts aérodynamiques, ce qui à priori devait favoriser la corrélation essai-calcul. La planche 8 montre les résultats obtenus en battement vertical dans un cas fortement chargé. L'évolution en envergure du crête à crête calculé est comparable à ce que donnent les résultats expérimentaux. Aucune comparaison valable n'a été obtenus en trainée, les caractéristiques de l'amortisseur de trainée utilisé étant mal connues.

La comparaison a été faite par ailleurs sur un rotor grandeur à pales plastiques. Il s'agit d'un rotor articulé en battement et semi-rigide en trainée. Les résultats expérimentaux proviennent d'enregistrements obtenus au cours de la mise au point du prototype. L'évolution en envergure des amplitudes des moments de battement est représentéesur la figure (9) et recoupe de manière satisfaisante les résultates Essais en Vol. L'analyse a été poussée jusqu'au contenu harmonique des contraintes pour deux sections de la pale situées à 0,28 R et 0,7 R (figure n° 10). La comparaison faite sur les cinq premières harmoniques montre que l'ordre de grandeur de chaque raie est correctement restitué, ce qui concorde avec la corrélation obtenue sur le crête à crête. La précision obtenue n'est pas toutefois suffisante pour une estimation du torseur des efforts d'excitation à la tête rotor. Une amélioration du calcul aérodynamique et l'introduction de modes propres couplés donneraient certainement des résultats plus précis pour ce problème particulier.

En ce qui concerne le cas de la trainée, le recoupement obtenu (figure nº 11) sur ce même rotor est nettement plus précis que pour lerotor maquette Modane.

#### 8 - CONCLUSIONS :

Nous estimons que ce genre de méthode est nécessaire pour permettre une estimation correcte et rapide des charges de dimensionnement sur rotor et moyeu. Les résultats obtenus aujourd'hui ne sont pas absolument parfaits, mais certaines idées ont permis sans trop de complexité supplémentaire, d'étendre le domaine de validité du calcul. Mais simultanément les méthodes faisant appel à de puissants moyens de calcul, doivent être développées, pour étudier les divers phénomènes, tester leur influence. De même l'analyse expérimentale fine du comportement du rotor doit nous permettre de progresser dans cette connaissance. La prédiction des charges de dimensionnement sur pale et moyeu devra se faire au niveau dessin, par une méthode simplifiée, faisant la synthèse des résultats obtenus par d'autres approches théoriques ou expérimentales.

# REFERENCES :

- 1 Ormiston R.A An actuator disc théory for rotor wake induced velocitées

  AGARD meeting Marseille september 1972
- 2 Pisiali R.A A method for predicting the aerodynamic loads and dynamic responses of rotor blades AD 628 583 janvier 1966
- 3 T.T Chang A method for predicting the trim constants and the rotor blade loadings and response of a single rotor helicopter USAAVLABS TN 67-71 novembre 67
- 4 Sadler G. Dévelopment and application of a method for predicting rotor free wake positions and resulting rotor blade airloads NASA CR 1911 decembre 1971
- 5 JJ. COSTES Calcul des forces aérodynamiques instationnaires sur les pales d'un rotor d'hélicoptère AGARD REPORT Nº 595 on unsteady aerodynamics of helicopter rotors
- 6 W.J Croskey Recent developments in rotor blade stall AGARD CP 111 september 1972
- 7 F.D Harris, F.J Tarsanin, R.K Fischer Rotor high speed performance, theory ws test

  Journal of the Américan helicopter Society Juillet 1970
- 8 W.H Tanner, Charts for estimating rotary wing performance in hover and at high forward speeds novembre 1964
- 9 M. Lecarme, Comportement d'un rotor au delà du domaine de vol usuel à la grande Soufflerie de Modane AGARD CP 111 - Septembre 1972
- 10 M. Lecarme et C. Armand, Essais de rotor d'hélicoptère dans la Grande Soufflerie de Modane 7è I.C.A.S Rome - Septembre 1970
- 11 R. Hirsch, Rotors d'hélicoptères DEP/ARA/NT/87/72
- 12 N.O Myklestad, Fundamentals of vibration analysis Mc Graw Hill Book Company
- 13 A.J Langrebe Simplified procedures for estimating flapwise bending moments on helicopter rotor blades WASA CR 1440 octobre 1969

FRINCIPC OF 18 METHODS DE CAICUL DE CONTRAINTES ROTOR
BIOCK DIAGREM OF STRESS ANALYSIS PROCRUM

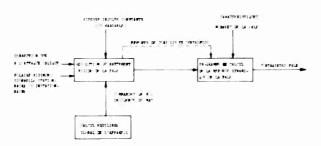
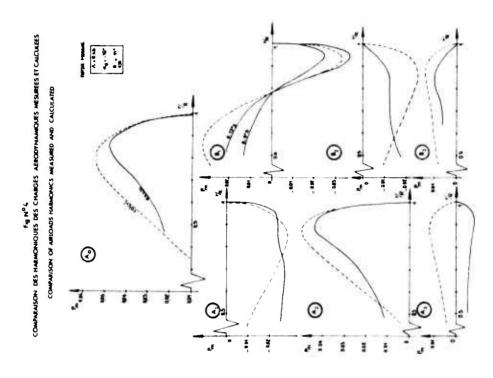


Fig N°2. ROTOR MAQUETTE DANS LA VEINE DE 5.1 MODANE MODEL ROTOR IN S.1 MODANE WIND TUNNEL





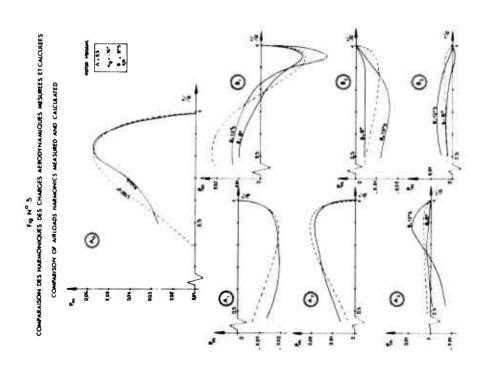


Fig. N° 5

3CHEMA SIMPLIFIE ADOPTE POUR LE SILLAGE
SIMPLIFIED SCHEMATIC ADOPTED FOR THE WARE MODEL

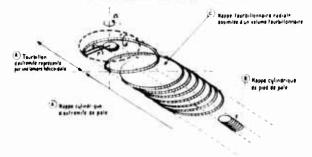


Fig Nº 6 PREDICTION DU CHAMP DES VITESSES INDUITES
APPENETION OF INDUIES VITESTES FALLO

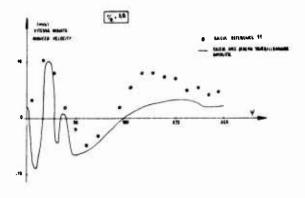


Fig. N° 7 COMPARAISON DES CHARGES AERODYNAMIQUES MESUREES ET CALCULEES AVEC. SCHEMA TOURBILLONNAIRE. SIMPLIFIE

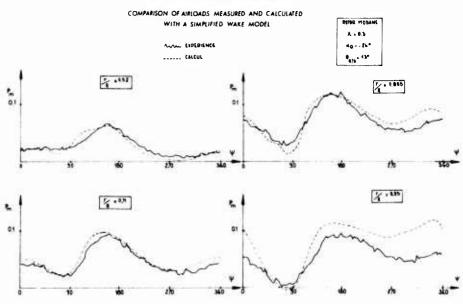
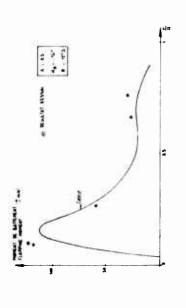


Fig.N." 6 ROTOR MODANE PREDICTION NES CONTRANTES DE BATTUMENT ROTOR MODANE PREDICTION OF FLADRIME STRESSES



ANALYSE HARMONIQUE DU MOMENT DE BATTEMENT HARMONE ANALYSES DE FLADPING MOMENT

F19 Nº 10

 $F_{ig}$  n° 9. Estimation as companits a battement surroung coanges calculate tarming embans of the transport of eight tests resalts

-

SANTAN A MARKET

,

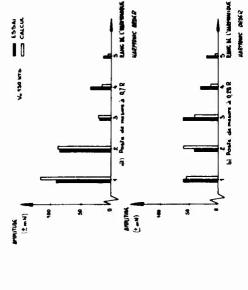
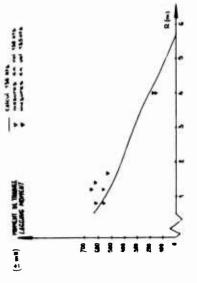


Fig. N° 11 Estimation des contraintes de Traime sur Rotor Crandica Calculate lageme moments combard to flight tests results



#### HELICOPTER ROTOR LOADS PREDICTION

bу

Peter J. Arcidiacono\* and Raymond G. Carlson\*\*

Sikorsky Aircraft
Division of United Aircraft Corporation
Stratford, Connecticut Scene
U. S. A.

SUMMARY

A review is presented of the assumptions and techniques forming the basis for detailed computation of rotor loads at Sikorsky Aircraft and the United Aircraft Research Laboratories. Typical correlation results showing the effects of variable inflow and unsteady aerodynamics on blade stresses and control loads are presented. These effects are shown generally to improve the accuracy of predicted results. A discussion of areas where further work can be expected to provide a stronger technical foundation for present analyses is presented. The principal areas include more detailed modeling of (1) the dynamic stall process, (to define unsteady drag, airfoil and blade sweep effects), (2) blade lifting surface effects (to model more accurately blade-vortex interaction effects) and (3) airframe dynamics effects (to define more accurately the dynamic coupling between blade and hub motions).

#### 1. INTRODUCTION

The problem of predicting the loads generated by rotating wings is, in many respects, one which is much more complex than that for fixed wings. The rotary wing problem is an aeroelastic one involving both unsteady stall and compressibility effects. The blade sweep angle also varies with time and, in addition, the flow field induced at the rotor is the result of a complex vortex wake whose treatment does not permit many of the simplifying assumptions so useful in fixed wing analyses. With the advent of high speed digital computing equipment, much progress has been made in developing improved analyses for predicting rotor loads. However, even with today's equipment, assumptions must still be made. Permissible assumptions, of course, depend on the rotor loads of interest. The rigor required in the analysis generally increases with the frequency of the loads that are to be predicted. The objectives of this paper are to (1) discuss the techniques and assumptions forming the basis of a current analysis in use at Sikorsky Aircraft, (2) present typical correlations with test data, and (3) indicate where further work may be expected to increase our capabilities.

# 2. REVIEW OF BASIC ELEMENTS OF THE PROBLEM

The basic elements that enter into the rotor loads problem are indicated in Fig. 1 and discussed below.

The problem is divided into two major areas: (1) the modeling of the aerodynamic forces acting on the blade and (2) the modeling of the blade response compatible with these forces. Because the typical rotor blade is a long flexible beam (and, in mary cases, is hinged at the root) its rigid body and elastic responses significantly affect the blade aerodynamic loading. Hence, there is a strong two-way coupling between the blade aerodynamic forces and its response.

The aerodynamic part of the problem is basically that of finding appropriate transfer functions which will operate on the relative motion of the blade with respect to the air to produce the aerodynamic forces acting on the blades. The problem is complicated by the fact that the forces being sought also contribute to the relative motion because of the associated blade response and wake vorticity which they produce. The blade response contribution to the relative blade-air motion is obvious and blade vorticity, of course, is related to flow field induced by the wake. Simultaneous solution of the blade force, blade response, and flow field equations presents an extremely difficult problem because of the many unknowns involved. This is particularly true if nonlinearities such as those associated with dynamic stall are to be considered. Usually some type of iterative procedure exploiting the weaker coupling mechanisms has to be employed. An approach frequently used is indicated in Fig. 1. Here the aerodynamic part of the problem is divided in o two parts. The first deals with modeling the effects of the trailing vorticity in the rotor wake to estimate the general flow field in which the blades operate. This flow field is based on an initial estimate of the blade loading distribution. The second part involves defining the forces acting on the blade that are compatible with this initial flow field estimate. This force definition can include nonlinear stall effects as well as blade response effects and, if desired, can be used to obtain a new estimate of the flow field for further iterations.

Modeling of the blade dynamic response, of course, involves solution of the blade equations of motion. The rigor of the solution will depend on the number of blade degrees of freedom treated and the extent to which interblade dynamic coupling effects due to control system and airframe motions are considered.

Figure 2 compares the assumptions made in a well known rotor analysis typical of the late fifties (Ref. 1) with those of an analysis currently in use at Sikorsky. The early analysis was oriented toward performance prediction and was limited primarily with regard to its treatment of rotor wake effects and the number of dynamic degrees of freedom considered. With the development of higher speed computing equipment, continual refinement in analytical techniques has been made. Typical analyses of today (also indicated in Fig. 2) incorporate some form of wake modeling, unsteady blade aerodynamics and an expanded number of dynamic degrees of freedom. Both of these analyses solve the overall problem through numerical

integration techniques, as this type of approach permits more rational treatment of the strong coupling between the blade aerodynamics and the blade response. Although most analyses use this general approach, there are still so many elements involved that further assumptions are required in handling these elements in order to produce a program that does not require an inordinate amount of computing time. The next section describes in more detail the particular approach currently in use at Sikorsky.

#### 3. SIKORSKY ROTOR AEROELASTIC ANALYSIS

A complete description of the analysis in this paper is not possible and the reader is referred to Refs. 2 to 4 for additional information. The intent here is only to indicate the major assumptions and solution techniques used. Only the steady flight version of the analysis is discussed.

A simplified block diagram of the analysis is shown in Fig. 3. Three basic programs are linked together. These are: the Blade Response Program, the Circulation Solution Program, and the Wake Geometry Program.

The Blade Response Program determines the fully-coupled response of a flexible rotating blade, given the distribution of the wake-induced velocities over the disc. The blade equations of motion are solved by expanding them in terms of uncoupled flatwise, edgewise and torsional blade modes. The modal technique facilitates the numerical integration of the blade equations by minimizing dynamic coupling terms. The basic differential equations of motion are documented in Ref. 2.

Two aerodynamic models can be used to determine the forces acting on the blades. Both assume two dimensionality. Reference 2 describes an early merodynamic model which is a conventional, quasi-steady aerodynamic model combined with the use of steady-state airfoil stall data. A model based on unsteady airfoil characteristics is described in Ref. 3. Typical unsteady lift and moment curves used for this model are shown in Fig. 4. These were obtained by generalizing data from tests of a two-dimensional airfoil executing prescribed sinusoidal motions. As indicated in Ref. 3, it has been assumed that the sinusoidal data could be generalized in terms of the section angle of attack, its first two time derivatives, and Mach number. Through this procedure, it is possible to apply the data to rotor blade operating conditions which involve translational velocity variations and multi-harmonic motions. Recent work by the United Aircraft Research Laboratories (Ref. 5) has shown that this assumption is reasonable. Blade section drag forces are evaluated using either steady-state data or, if desired, by a procedure which allows "unsteady" drag to be synthesized from the unsteady lift data. The synthesization procedure is based on the degree to which the unsteady lift departs from potential flow values (see Ref. 4). No attempt to account for any radial flow effects on lift or moment has been made in the analysis. Although such corrections have been proposed in the literature, the underlying data are extremely questionable, as will be discussed later. With either the quasi-steady or the full-blown unsteady aerodynamic model, the equations of motion are numerically integrated until a converged cyclic motion of the blade is achieved that is compatible with the prescribed induce! velocity distribution.

The function of the Circulation Solution Program is to compute a rotor circulation distribution that is compatible with a prescribed set of blade section operating conditions and a prescribed rotor wake geometry. Once the circulation distribution is known, the induced velocity distribution over the rotor follows immediately. This induced velocity distribution can be used to update the original input to the Blade Response Program. An iteration is performed between the programs to assure compatibility of the induced velocities and the blade aerodynamic and dynamic boundary conditions. The general technical approach used in the Circulation Program is basically similar to that of Ref. 6 and represents a rotary-wing equivalent of the classical lifting-line approach used successfully for fixed wings. Two major differences from the Ref. 6 approach are: (1) the elimination of the shed vorticity elements in the wake (i.e. elements arising from time variations of blade bound vorticity) and (2) the inclusion of unsteady effects on local lift curve slope, blade section angle-for-zero lift and blade stall angle. Although the former modification technically violates the Helmholtz law, it is believed that a more accurate representation of the shed wake effects is obtained through the use of the unsteady airfoil data in the Blade Response Program. This implies that the primary effects of the shed vorticity are those associated with the wake region near the blade and, thus, can be approximated by those of a fixed-wing type of wake. Miller (Ref. 7) shows that this is reasonable at rotor advance ratios usually of interest. This approach not only permits a factor of two reduction in the computing time of the Circulation Program, but more importantly, also permits nonlinear unsteady stall effects to be included in a rational way in the Blade Response Program.

The Circulation Program requires that the wake geometry be specified a priori. In lieu of more precise information, the assumption of a classical hondistorted wake defined from momentum considerations has usually been made. Recently, methods for computing more representative wake geometries have become available. The approach developed by Landgrebe (Ref. 8) is employed in our analysis. The approach is straightforward if the wake circulation distribution is prescribed. At a given instant of time, the biot-Savart law is used to determine the wake self-induced velocities at various control points in the wake. These velocities are integrated over a small time interval to define a new wake geometry, following which new self-induced velocities can be computed. The process of alternately computing new velocities and wake geometry is continued until a converged, periodic distorted wake geometry is reached. By dividing the wake into near and far regions relative to each control point in the wake, an approximate and cost-effective, analysis is obtained. Only the tip vortex is allowed to distort as it represents the dominant wake element. An iteration between the Circulation Program and the Wake Geometry Program is required to assure compatibility of the final circulation distribution and wake geometry.

It should be evident from the preceding discussion that this approach to solving the combined airload-flow field problem can only be applied to steady operating conditions. Also, there are many ways to cycle through the programs, and the convergence of the iteration procedure will depend on the strength of the coupling between the various elements of the problem. We have generally found that convergence of the airloads is rapid and that only one pass through the Circulation and Wake Geometry Programs is required. To control computing time, the iteration procedure is set up to minimize the number of passes through the

longest running program, which in this case, is the Wake Geometry Program. Typical running time for the three programs on the UNIVAC 1108 computer are:

Blade Response Program 2 minutes Circulation Solution Program 1 minute Wake Geometry Program 7 minutes

Our iteration procedure generally involves 2 passes through the Blade Response Program and 1 pass through the Circulation Program when a classical wake geometry is used (a total of 5 minutes). If the Wake Geometry Program is used, one pass through it is used together with one additional pass through the Circulation Program for a total of 13 minutes.

#### 4. TYPICAL COMPARISONS OF PREDICTED AND MEASURED RESULTS

Correlation studies using the Sikorsky Rotor Aeroelastic Analysis have demonstrated the importance of including both the variation in induced velocities (or inflow) over the rotor disc and unsteady aero-dynamics in calculating blade stresses and control loads. This is particularly true in the high speed, high loading flight conditions where the rotor is in stall over part of the disc. It is such conditions that determine the operational limits on the rotor system structural capability. Therefore, it is these conditions that have received most of the attention for correlation studies. The bulk of the results presented in this paper are based on a classical, nondistorted wake geometry inasmuch as wake geometry effects are still largely under evaluation. All results are also based on the use of a steady-state drag model.

Some recent results which point out the significance of variable inflow and unsteady aerodynamics are shown in Figs. 5 through 8. The flight condition corresponds to an advance ratio of 0.36 and blade loading of 0.076. (All results have been nondimensionalized by the measured amplitudes. In Figs. 5 and 7 the measured amplitudes selected are those at stations at 69% radius and 18% radius, respectively.) As indicated in the figures, better correlation of flatwise vibratory stresses has been obtained by including these phenomena. Amplitude, or half-peak-to-peak value, of flatwise stress is plotted against radius in Fig. 5. As shown, the use of an assumption of a constant inflow over the rotor disc with steady-state aerodynamics produces predicted stress of about 80% of the measured peak value, with the maximum occurring too far inboard. Adding the effect of inflow variations but retaining steady-state aerodynamics had little effect on the amplitude, but did shift the peak radially outboard. The further addition of unsteady aerodynamics, in this case using the method of Ref. 3, increased the amplitude to about 90% of the measured flatwise vibratory stress. Consideration of the azimuthal variation (i.e. time history) of flatwise stress at 60% radius, the point of maximum measured stress, further emphasizes the improvement obtained when variable inflow and unsteady aerodynamic effects are included in the analysis (Fig. 6). The constant inflow, steady-state aerodynamic analysis shows predominantly a one-per-rev response with little evidence of the higher harmonic content present in the test data. The inclusion of variable inflow, which provides significant higher harmonic loading of the blade, reproduces more closely the observed variations in stress around the azimuth. Unsteady aerodynamics result in an increase in the stress amplitude but has little further effect on the general predicted azimuthal signature.

The achievement of good correlation of edgewise vibratory stresses has generally proven a more difficult task than that for flatwise stresses. Edgewise stresses for the same condition as shown in Figs. 5 and 6 are given in Figs. 7 and 8. Some improvement in the amplitude of response is shown with the successive addition of variable inflow and unsteady aerodynamics. Comparison of the azimuthal variations at 50% radius show the same trends found for the flatwise stresses. With constant inflow and steady state aerodynamics, the higher harmonic content of the edgewise stress is low. The addition of variable inflow increases the higher harmonic content, in particular the four-per-rev component of edgewise stress, which is dominant due to the location of the first edgewise flexible mode near four-per-rev. Unsteady aerodynamics serve to increase the amplitude a bit more, again without changing the character. The principal area of discrepancy appears on the retreating blade. This is the area that would be expected to be influenced by the modeling of unsteady drag effects.

The torsional response of the blade, as measured by the push rod loads, was over-predicted using the combination of variable inflow and unsteady aerodynamics. In this case the best correlation, as shown in Fig. 9, was obtained using variable inflow without unsteady aerodynamics. It has been found that the inclusion of unsteady aerodynamics is important in sustaining the type of torsional oscillation, over the retreating half of the disc, which is generally attributed to stall flutter. Figure 10, for a different aircraft and rotor system, shows the kind of correlation being obtained in flight conditions in which a build up occurs in control loads due to the high frequency oscillations. The inclusion of variable inflow in the analysis produces a variation in air loading causing torsional oscillations of the advancing blade. The inclusion of unsteady aerodynamics sustains and amplifies this oscillation over the retreating half of the disc, due to the negative torsional damping effects produced by dynamic stall hysteresis effects. This is demonstrated in Fig. 11, where the analytical curve of Fig. 10 is reproduced, along with analytical results using variable inflow but no unsteady aerodynamics. The decay in the oscillation when unsteady aerodynamics are neglected is most evident.

As mentioned previously, the results presented in Figs. 5-11 use a variable inflow model which is based on a classical, non-distorted, helical rotor wake geometry. Wake distortion effects on rotor vibratory loads have not been systematically analyzed at this time. The study reported in Ref. 4 indicated that wake distortions had little effect on integrated rotor loads, as might be anticipated. However, Fig. 12, which is reproduced from Ref. 4, indicates that significantly different azimuthal variations in local blade loadings can result when wake distortion effects are included in the analysis. It would appear that inclusion of such effects will be necessary for accurate prediction of rotor vibratory forces. Also, wake distortion effects at low speeds and hover are dominant. At the present time, further work is required to verify that wake distortions at high flight speeds are equally significant and, thereby, justify the relatively large increase in computing time entailed in their computation.

4-4

#### 5. TECHNOLOGY AREAS REQUIRING FURTHER DEVELOPMENT

The preceding section has indicated the type of correlation that has been achieved using current technology. Those aspects of the analysis which can be improved through further development are discussed below.

#### 5.1 Dynamic Stall Modeling

It is well known that retreating blade stall is the prime factor limiting the forward flight capabilities of rotating wing aircraft. Because it is a relatively local effect, the ultimate capability of a given aircraft is usually dictated by the degree to which stall can be penetrated before such stall-related effects as power losses, increased vibration or control loads, etc. become intolerable. As a result, the rotary wing designer is relatively more concerned with the details of stall aerodynamics than is his fixed wing counterpart. Rotor stall is a complex unsteady aerodynamic phenomenon. Although much progress has been made in improving the representation of unsteady effects in our analyses, additional work is required before reliable computations of rotor stall characteristics can be routinely made in the design process.

Most work in the modeling of dynamic stall has been oriented toward applying two-dimensional data from a sinusoidally oscillating airfoil. To apply such data to the rotor problem requires some means of generalizing the data so that the aerodynamics of more arbitrary airfoil motions can be predicted. It would also be highly desirable to find some reliable, practical method for handling different airfoil contours so that the need for expensive wind tunnel tests could be minimized. Two methods for generalizing unsteady data to arbitrary motions and different airfoils are found in Refs. 3 and 9. Figure 13 presents an interesting comparison of results predicted by each of these methods for ramp-type variations in angle of attack. Also shown in the figure are experimental data obtained from Refs. 10 and 11. Distinctly different trends are noted for the measured ramp characteristics and those predicted from sinusoidal data using the methods of Refs. 3 and 9. The measured ramp data indicate a relatively linear increase in maximum lift, whereas the predictions show a diminishing effectiveness of angular rate in increasing lift capability. This is believed to be due to the fixed amplitude oscillations which were used in the sinusoidal tests. In addition, the predictions individually differ significantly with regard to the increase in maximum lift coefficient produced by a given angular rate.

To explore this situation further, Sikorsky Aircraft has been developing an alternate and, hopefully, simpler approach to the generalization problem. The model is based on the hypothesis that flow separation will not occur until a critical, nondimensional time interval is exceeded after the static stall angle of attack has been reached. The critical time interval ( $\Delta t$  separation) is given by

$$\tau^* = \frac{\text{U}\Delta t}{\text{separation}}$$

Here U and c are the translational velocity and chord of the airfoil, respectively. The quantity,  $\tau^*$  is, thus, equivalent to the number of chordlengths traveled by the airfoil during the critical time period. Analysis of available data for sinusoidal motion indicates a  $\tau^*$  of about 6 applies (Fig. 14). When this value for  $\tau^*$  is applied to a ramp-type motion, a linear relation between the increase in maximum lift coefficient and the angular velocity parameter  $\underline{(\dot{c}\dot{\alpha})}$  is found

$$\Delta C_{lmax} = 12 \frac{(ca)}{U}$$

the complete Navier-Stokes fluid equations.

Here  $\dot{\alpha}$  is the time rate of change of airfoil angle of attack in radians/sec units. This relation is also shown in Fig. 13 and is seen to agree qualitatively quite well with the experimental results.

The techniques discussed above for modeling dynamic stall are rather pragmatic ones which do not address the details of the dynamic stall process. More fundamental analyses are now being pursued, and with the advent of higher speed computing equipment can be expected eventually to become practical approaches to the problem. An initial analysis of this type is described in Ref. 12. However, the sample application of the analysis to a ramp motion in Ref. 12 resulted in a much lower increase in lift (0.3 for  $c\alpha = .048$ ) than has been either observed or predicted from other less fundamental approaches. This inadequacy can be expected to be eliminated by more accurately modeling the viscous flow aspects of the problem through solution of

The conclusion to be drawn from Fig. 13 is, then, that techniques for predicting the dynamic stall of an airfoil undergoing relatively arbitrary motions require further development and substantiation.

The preceding discussion has been concerned with generalizing dynamic stall characteristics for a given airfoil. Another aspect of the problem, of course, is the need to generalize the effects of airfoil contour. The available methods (Refs. 3 and 9) tend to assume that the unsteady effects are approximately the same for all airfoils and can be accounted for if the static stall characteristics are known. Further work is required to verify this contention.

All experimental dynamic stall work to date has also emphasized nonrotating, two-dimensional, upswept airfoil models. Thus, radial flow effects due to blade sweep, centrifugal effects and spanwise pressure gradients on unsteady characteristics have been neglected. There appears to be a general consensus that centrifugal force effects are negligible (see Refs. 13-17 and Fig. 15), at least for the high velocity sections of typic leaders. The effect of blade sweep angle is less well defined but is potentially important. Fixed wing experience has shown that inboard sections of sweet back wings can sustain much higher lift coefficients than would be estimated on the basis of pressure gradients taken normal to the span. However, test data are not available for the range of sweet angles, aspect ratios and spanwise pressure gradients typical of rotor blades. Attempts have been made to use available fixed-wing static data to derive two-dimensional spanwise flow corrections (see Ref. 18). These data (presented in Ref. 19) were

56

obtained by measuring total forces on a swept, finite-span wing. Since the wing sweep angle was varied by yaving the wing about its mid-span point, one half of the wing was swept aft - certainly not an ideal situation. Analysis of the data of Ref. 18 shows a strong finite-span effect (see Fig. 16) and the applicability of the data to the rotor blade situation is questionable. It would appear that a truly two-dimensional, swept wing would show no enhancement of lift capability. Hence, the meaning of the "two-dimensional" corrections derived in Ref. 18 is not clear. Appropriate tests to clarify the situation and to introduce other variables such as spanwise pressure gradient should be conducted. Finally, it should be mentioned that a big gap in knowledge exists with regard to the unsteady drag characteristics of rotor blades.

While these fundamental questions are being pursued, the designer requires dynamic stall information now. To fill this need, the effects of more common design parameters on the dynamic stall-stall flutter process are being studied experimentally by Sikorsky and UARL. Figure 17 illustrates the basic approach being used. The model is a two-dimensional airfoil mounted on bearings and driven in a sinusoidal pitch motion by the linkage shown. By properly selecting the inertial characteristics of the airfoil and the stiffness of the torsional flexure, which is in series with the oscillating mechanism and the airfoil, it is possible to simulate the dynamic equation of motion of the first torsional mode of the blade. The blade is oscillated sinusoidally at a frequency and amplitude which simulates the basic once-per-revolution (IP) angle of attack oscillation experienced by a rotor blade in forward flight. If the system is susceptible to stall flutter, high frequency torsional oscillations will occur. Such have been observed in initial tests (see right hand panel of Fig. 17) and we are now investigating the effects of system design parameters on stall flutter. These include airfoil shape, and torsional inertia, stiffness and frequency. Figure 18 shows some early results indicating improvement in angle of attack capability produced by a Sikorsky airfoil design as compared to that for a NACA 0012 airfoil. This increase tends to correlate with the increase in static lift capability of the Sikorsky airfoil.

The influence of spanwise flow on rotor blade stall characteristics is also being investigated by Sikorsky. For this purpose, new experimental techniques for acquiring two-dimensional airfoil data are being developed. A unique airfoil dynamic force measuring system is being used which will minimize wind tunnel wall corrections and provide an initial capability for measuring unsteady drag. These techniques will be used to provide better data on the following:

- 1) the effects of high Mach number (M 1.0),
- 2) the effects of sweep on steady stall characteristics,
- 3) the effects of sweep on unsteady stall characteristics.

Tests have already been conducted in the first area and initial tests in the remaining areas should be started in late 1972.

#### 5.2 Lifting Surface Effects

One of the principal assumptions made in the current analysis is that the blades are lifting lines. While this assumption is valid for many regions over the rotor disc, it is not valid near the blade tips and in those areas where a blade passes close to a vortex in the rotor wake. An example of the type of differences in loading that are predicted by lifting line and lifting surface theory is shown in Fig. 19. The lifting surface theory is being developed by Adamcyzk of the United Aircraft Research Laboratories. The results of Fig. 19 show the time histories of lift and moment coefficients generated by a blade as it encounters a series of two-dimensional vortices. Lift coefficient variations are greatly overestimated by the lifting line theory while the pitching moments are underestimated. The variations in pitching moment would seem to be particularly important because of the torsional flexibility of typical rotor blades. Effects such as these are relatively local ones; however, their inclusion would appear to be necessary before the detailed loadings needed for accurate calculation of rotor vibratory forces can be obtained. Work is proceeding to develop the theory further and place it in a form suitable for use in rotor analyses. This is being complemented by Sikorsky work to define characteristics of vortices having circulation strengths typical of full scale blades (Ref. 20).

#### 5.3 Transient Wake Effects

As mentioned previously, the rotor loads analysis described in the earlier section of this paper is basically a steady flight analysis. This limitation arises because of the way in which the three basic elements of the problem are coupled in the solution. Solutions for blade response and the flow field are obtained assuming them to be semi-independent problems. This invalidates the analysis for the calculation of either aircraft transient flight characteristics or blade stability characteristics where the flow field is expected to be an important factor. Transient flight loads analyses are available, but these either neglect wake effects entirely or treat them in a quasi-steady fashion. There is no technical reason why the various elements of the overall problem cannot be more closely coupled so that these limitations can be eliminated. Computing time has represented the principal consideration to date, and it may be expected to be eliminated as a factor with newer generations of computers that will inevitably be developed.

# 5.4 Airframe Dynamics Effects

In seeking ways to improve the prediction of vibratory loads on a helicopter, attention will also have to be focused upon the dynamic coupling of the rotor and airframe. Airframe dynamic response effects may prove to be significant for accurate prediction of vibratory loads imposed on the fuselage. These effects can enter in two ways. First, coupling of the airframe modes and blade modes can occur, producing shifts in the blade natural frequencies. This will alter the predicted blade dynamic response, and therefore, the blade vibratory loads. Since for steady state flight the hub motions are predominantly at the principal excitation frequency (given by the number of rotor blades times the rotor speed), it is precisely those frequencies of most concern for vibration which are affected by the hub motion. Secondly, the motion of the airframe at the hub also produces motion of the rotor blades, which, strictly speaking, changes the airloads. However, these changes in airloads should be small since the vibratory hub motions are small.

The airframe-rotor dynamic coupling should become more significant as the rigidity of the blade attachment to the airframe increases. Articulated rotors should exhibit less coupling effects than a non-articulated rotor with a stiff root restraint. The Sikorsky Rotor Aeroelastic Analysis presently only includes rigid body airframe degrees of freedom. The inclusion of flexible airframe modes should be considered in the future whenever aerodynamic modeling techniques have progressed to the point where accurate computation of rotor vibratory loads appears feasible.

Finally it should be noted that the airframe flexibility and airframe modes can have a significant effect on the stability of a rotor system. Such effects are, or course, included in rotor stability analyses.

#### 6. CONCLUDING REMARKS

This paper has been devoted to a discussion of the analytical rotor load techniques in current use and being developed at Sikorsky Aircraft and the United Aircraft Research Laboratories. The past decade has seen significant progress in the development of rotor flow field models and in the treatment of unsteady aerodynamic effects. Although further developments are necessary to provide a more thorough understanding of the dynamic stall process, sufficient progress has been made to warrant a more in depth assessment of the effects of design parameters on rotor stall-related operating limits. Techniques for predicting wake geometry have been developed and remain to be applied in detail. Spanwise flow and lifting surface refinements are less well developed, but are being vigorously pursued. Unsteady drag information is almost totally nonexistent and is certainly needed. Advances in computing machinery will make possible more clos'ly coupled solutions of the blade response - flow field problem for application to maneuver and rotor statility problems, and, ultimately, analytic treatment of the detailed viscous flow processes governing rotor stall should also be possible. However, it should be emphasized that while we make every attempt to improve the technical foundation of our analyses, we should make corresponding efforts to remember the ultimat; user of the analysis--the designer. He wants a thoroughly checked out analysis, whose outputs have been digested and cast in the simplest, most fundamental form and provide him with easy-toobtain, cost-effective answers. Too often, analyses never are taken beyond the research stage with the result that they are often not acceptable for use in the practical world.

#### 7. REFERENCES

- Tanner, W. H., Charts for Estimating Rotary Wing Performance in Hover and at High Forward Speeds, NASA CR-114, 1964.
- 2. Arcidiacono, P.J., Prediction of Rotor Instability at High Forward Speeds, Vol. 1: Differential Equations of Motion for a Flexible Helicopter Rotor Blade in Steady Flight Including Chordwise Mass Unbalance Effects, USAAVLABS Technical Report 68-18A, February 1969.
- Arcidiacono, P. J., et al., Investigation of Helicopter Control Loads Induced by Stall Flutter, USAAVLABS Technical Report 70-2, March 1970.
- 4. Bellinger, E.D., Analytical Investigation of the Effects of Blade Flexibility, Unsteady Aerodynamics, and Variable Inflow on Helicopter Rotor Stall Characteristics, NASA CR-1769, September 1971.
- 5. Carta, F.O., et al., Determination of Airfoil and Rotor Blade Dynamic Stall Response, 28th Annual National Forum of American Helicopter Society, Preprint Number 613, May 1972.
- 6. Piziali, R.A., and DuWaldt, F.A., A Method for Computing Rotary Wing Airload Distributions in Forward Flight., TRECOM Report No. TCREC TR 62-44, November 1962.
- 7. Miller, R.H., Unsteady Airloads on Helicopter Rotor Blades, Journal of the Royal Aeronautical Society, Vol. 68, April 1964, pp. 217-229.
- 8. Landgrebe, A. J., Analytical Methods for Predicting Rotor Wake Geometry, Journal of the American Helicopter Society, Vol. 14, No. 4, October 1969, pp. 30-32.
- Gross, D., and Harris, F., Prediction of In Flight Installed Airloads From Oscillating Airfoil
  Data, paper presented at the American Helicopter Society 25th Annual National Forum, Washington,
  D.C., May 1969.
- 10. Fung, Y.C., The Theory of Aeroelasticity, John Wiley and Sons Inc., New York, 1955.
- 11. Ham, N. D., and Garelick, M.S., Dynamic Stall Considerations in Helicopter Rotors, Journal of AHS. October 1967.
- 12. Crimi, P., Development and Application of a Method for Analyzing Dynamic Stall of Helicopter Rotor Blades, Paper presented at Working Meeting on Dynamic Stall and Stall Flutter, Langley Research Center, NASA, July 1971.
- 13. Dwyer, H.A., and McCroskey, W.J., Crossflow and Unsteady Boundary Layer Effects on Rotating Blades, AIAA Paper 70-50, January 1970.
- 14. Young, Jr., W.H., and Williams, III, J.C., The Boundary Layer on Rotating Blades in Forward Flight, AIAA Paper 70-49, January 1970.

- McCroskey, W.J., and Dwyer, H. A., Methods of Analyzing Propeller and Rotor Boundary Layers, NASA SP-228, 1969.
- 16. Himmelskamp, H., Profiluntersuchungen an Einem Umlaufenden Propeller, Diss. Göttingen 1945, Reports of the Max-Planck-Institut für Stromungs forschung, Göttingen, No. 2, 1950.
- Betz, A., Höchstauftrieb von Flügeln an Umlaufenden Rädern, Zietschrift für Flugwissenschaften, Vol. 9, No. 4/5, April-May 1961, pp. 97-99.
- Harris, F.D., et al., Rotor High Speed Performance, Theory Versus Test, Proceedings of the V/STOL Technology and Planned Conference, AAFDL, September 1969.
- Purser, P.E., and Spearman, M. L., Wind-Tunnel Tests at Low Speed of Swept and Yawed Wings Having Various Plan Forms, NACA TN 2445, December 1951.
- 20. Rorke, J.B., and Morifitt, R.C., Wind Tunnel Simulation of Full Scale Vortices, Paper presented at the 28th Annual National Forum of the American Helicopter Society, Washington, D.C., May 1972.

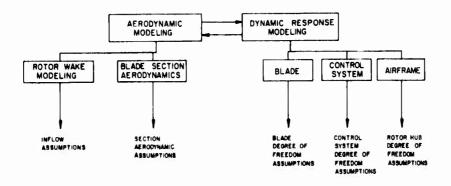


FIGURE I BASIC ELEMENTS OF ROTOR LOADS PROBLEM

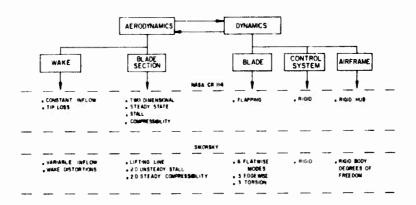


FIGURE 2 COMPARISON OF ASSUMPTIONS OF TWO ROTOR LOADS ANALYSES

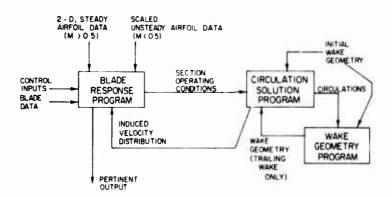
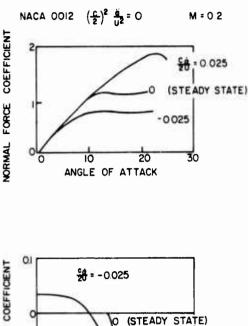


FIGURE 3 CURRENT SIKORSKY ROTOR LOADS ANALYSIS



O (STEADY STATE)

O (O (STEADY STATE)

FIGURE 4 TYPICAL GENERALIZED UNSTEADY DATA

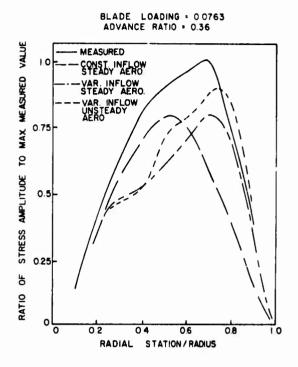


FIGURE 5 EFFECT OF VARIABLE INFLOW AND UNSTEADY ALRODYNAMICS ON BLADE FLATWISE STRESS AMPLITUDE

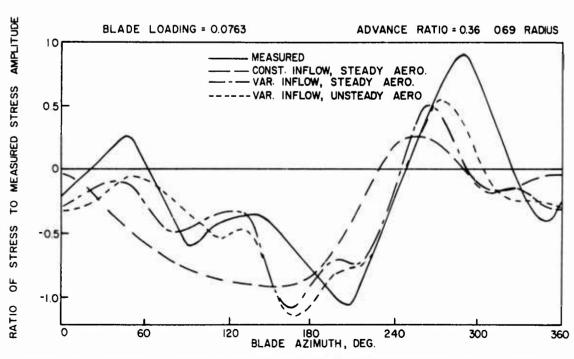
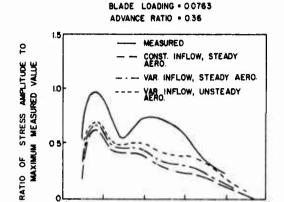


FIGURE 6 EFFECT OF VARIABLE INFLOW AND UNSTEADY AERODYNAMICS ON FLATWISE STRESS TIME HISTORY



04

00

0.2

FIGURE 7 EFFECT OF VARIABLE INFLOW AND UNSTEADY AERODYNAMICS ON EDGEWISE STRESS AMPLITUDE

RADIAL STATION / RADIUS

06

08

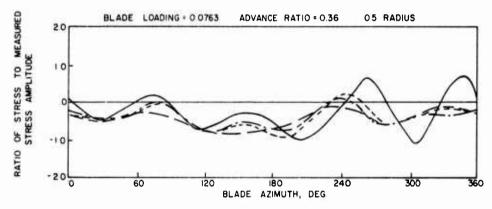


FIGURE 8 EFFECT OF, VARIABLE INFLOW AND UNSTEADY AERODYNAMICS ON EDGEWISE STRESS TIME HISTORY

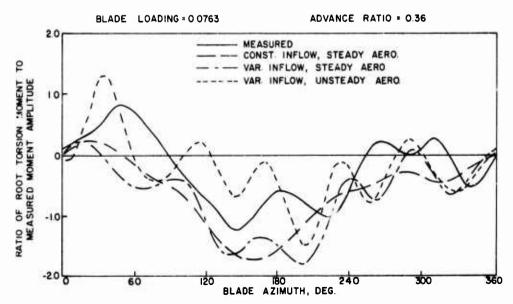


FIGURE 9 EFFECT OF VARIABLE INFLOW AND UNSTEADY AERODYNAMICS ON ROOT TORSION MOMENT TIME HISTORY

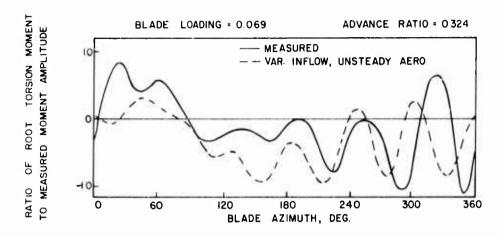


FIGURE 10 EFFECT OF VARIABLE INFLOW AND UNSTEADY ON ROOT TORSION MOMENT TIME HISTORY

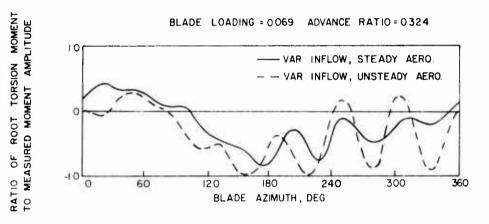
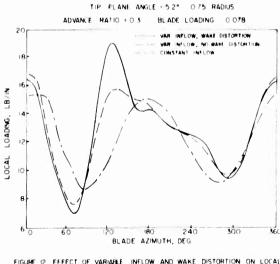


FIGURE II COMPARISON OF PREDICTED ROOT TORSION MOMENT WITH AND WITHOUT UNSTEADY AERODYNAMICS



EFFECT OF VARIABLE INFLOW AND WAKE DISTORTION ON LOCAL BLADE LOADING FIGURE 12



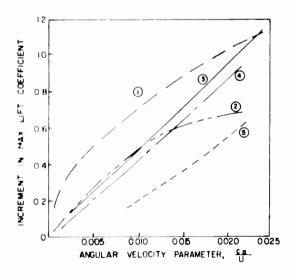
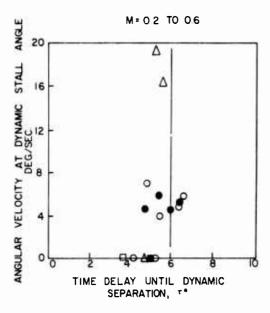


FIGURE 13 COMPARISON OF MEASURED RAMP LIFT CHARACTERISTICS WITH THOSE PREDICTED FROM GENERALIZED PROCEDURES

BASED ON RESULTS FROM REFS 16 AND 17





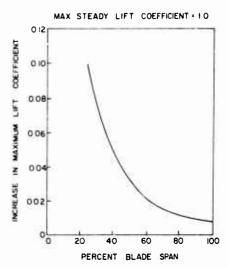


FIGURE 14 CORRELATION OF TIME DELAY WITH ANGULAR VELOCITY

FIGURE 15 INCREASE IN MAXIMUM LIFT DUE TO CENTRIFUGAL FORCE

2 DIMENSIONAL THEORY (COS<sup>2</sup> A)

O DATA, REF 18, ASPECT RATIO = 6

△ DATA, REF IS, ASPECT RATIO = 3

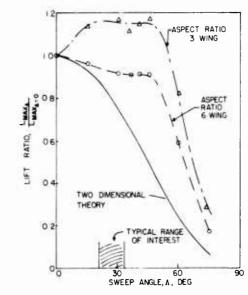


FIGURE 16 COMPARISON OF LIFT CAPABILITY OF FINITE-SPAN SWEPT WINGS WITH THEORY

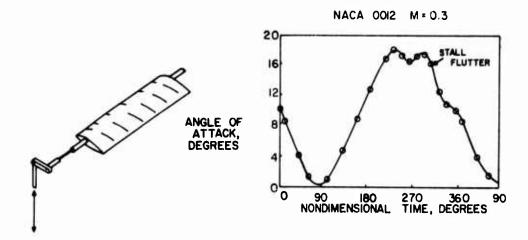


FIGURE 17 STALL FLUTTER MODEL AND TYPICAL DATA

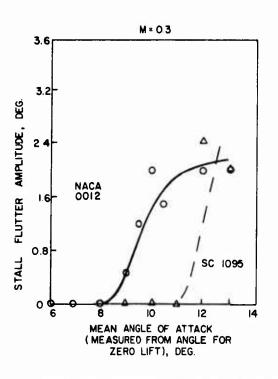


FIGURE 18 COMPARISON OF STALL FLUTTER RESPONSES FOR TWO AIRFOILS

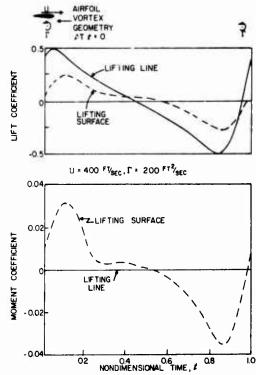


FIGURE 19 COMPARISON OF LIFTING LINE AND LIFTING SURFACE RESULTS FOR TYPICAL VORTEX ENCOUNTER

ASSESSMENT OF ROTOR LOADS PREDICTION TECHNIQUES USED BY SIKORSKY AIRCRAFT CORPORATION AND AEROSPATIALE

bν

John L. Shipley
Chief, Army Aeronautical Research Group
Langley Directorate
US Army Air Mobility Research and Development Laboratory
Mail Stop 124
NASA-Langley Research Center
Hampton, VA 23665

I have chosen to separate my comments into three sections, the first being some general comments about each author's program, comments on the philosophy of model development employed and then areas which I think we need to expand the current capability in predicting rotor loads.

As Mr. Gallot has stated, his analysis is based on judgment of accuracy versus run time and complexity with a specific objective that the resulting program can be easily used by the designer. The desire for a simplified method for accurately calculating rotor loads is universal and we all share this with Mr. Gallot. The resulting program is to be used only for blades with high torsional stiffness operating below the onset of stall so that pitching moments will produce small oscillations when compared to collective inputs. Within these constraints, the program should be adequate for estimating structural strength requirements, as was the intent of the program. However, I think that Mr. Gallot will agree that some additional refinements, such as unsteady aerodynamics and iteration between blade response and aerodynamics, should be considered before the results could be used for such things as fatigue design loads.

The analytical results predicted by the author's program shows good correlation with experimental data. However, the experimental data used in the comparison were for conditions of tip path plane angles of 16 and 24 degrees and advance ratios greater than 0.3. These correspond to flight conditions where the free stream contributions to the in-flow significantly reduce the wake vorticity effects. It would be interesting to see if lowering the tip path plane, through reduced fuselage drag, would perhaps reintroduce higher harmonic forces of sizeable magnitude.

In regard to Mr. Arcidiacono's paper, he has made what appears to be a reasonably successful attempt to include the majority of those parameters which have, over the past five years, come to be associated with improved prediction of rotor loads. Although they may be a little optimistic, run times of the order of magnitude cited by Mr. Arcidiacono are certainly well within reason for a program which includes coupling of the blade aerodynamics and dynamics, wake geometry, and unsteady aerodynamics.

As is the case with several of the programs to be described in this meeting, the Army has on occasion used a version of the rotor loads program described by Mr. Arcidiacono. 'Once the initial problems, always encountered when the unfamiliar attempt to use a fairly complex computer program, were overcome, reasonably good correlation of results have been obtained, especially when the unsteady aerodynamic routine was used.

Even though the phenomena is not fully understood and analytical techniques, which model all aspects of the problem, are not available, the results justify including unsteady aerodynamics. Most authors will agree that if the results are going to be used for determining fatigue life and sizing of critical components at highly loaded conditions, some attempt at representing unsteady aerodynamics should be made. However, there is still a long way to go before the claim can be made that a full blown unsteady aerodynamic model is available, especially in the areas of unsteady drag and different airfoil contours.

The discussion on unsteady aerodynamics recalls to mind Frank Harris' mid 1960 paper, "Blade Stall, Half Fact, Half Fiction", in which he was, to my knowledge, the first advocate for treating lift, drag, and moment stall as three separate phenomena, which may or may not occur simultaneously. Based on Mr. Arcidiacono's results showing good correlation of the amplitude and duplication of the forced oscillation, it appears that the lift and pitching moment portion of the dynamic stall process can be handled reasonably well analytically; that is, as long as the analysis is confined to a symmetrical 0012 airfoil for which oscillating data are available. It should be emphasized that the capability described does not include treatment of different airfoils, while other analyses are limited by the assumption that the dynamic stall process for all airfoils is identical.

Recalling the funds and time expended on obtaining the oscillating 0012 airfoil data, one has to agree with Mr. Arcidiacono's statement that a more practical and economical method has to be developed for treating unsteady pitch and lift characteristics. Considering the wide variety of new generation airfoils currently being developed for rotor applications, there are not enough funds or tunnel facilities available to obtain the experimental data necessary for the current analytical procedures.

While on the subject of experimental data, figure 13 of the paper, which is a comparison of measured ramp lift characteristics with those predicted from generalized procedures, causes some concern. There seems to be more discrepancy between Fung and Ham's experimental data than there is between the theories. Improvement in the method of generalizing unsteady data by the author's approach is obtained only when Ham's data are disregarded. This also points out another problem area in unsteady aerodynamics, which is the wide discrepancy in unsteady airfoil data which still has not been resolved. Everyone has his own individual set of data which he believes is the only valid one in existence.

Mr. Arcidiacono has adequately covered the inability to analytically handle unsteady drag in current analysis. Not only are analytical techniques lacking, but to date no reliable experimental unsteady drag data has been obtained. This degrades from the accuracy of the inplane rotor loads predictions, as shown in the paper, and can also significantly alter the results of subsequent rotor stability analysis if air

resonance is a potential problem.

One additional point which is applicable to both papers concerns uncoupled modes. The effects on accuracy are, of course, a function of the degree of the coupling. While in the past this may not have been a significant parameter, some of the new composite or fiber blades have large amounts of coupling between the flap and torsion modes which should be considered in the analysis.

I would next like to discuss briefly the apparent philosophy used by the authors in developing their math models as seen from a novice user's viewpoint. One's initial reaction is that the authors employed opposing approaches to the problem of calculating rotor loads and stresses; one approach being that increased complexity and computer time is not warranted by the resulting accuracy, while the other approach considers increased accuracy the prime motivation. However, both have discussed a math model which was developed with different objectives or application of results intended. I think that there is a requirement for both types of models and the authors have employed an underlying common philosophy in math modeling which is reasonably sound and warrants a few minutes of discussion.

As mentioned previously, Mr. Gallot has taken what could be called a cost effectiveness approach wherein complexity and usability are prime considerations. Starting with a definite objective for which the results are intended to be used, that is, by a design engineer to size rotor components, he has apparently determined the minimum accuracy which he considers acceptable. Within these constraints he then makes simplifying assumptions which are used as the basis for establishing the resulting range of rotor characteristics and operating conditions for which the results are applicable.

Whereas the analysis described by Mr. Arcidiacono is developed along the lines of a model whose results would be applicable for a wider range of conditions and could be used in the majority of situations when detailed rotor loads are required, almost a quote "universal model" approach. As a result, improved accuracy of the results are prime objectives. !!ence, increased program complexity and run time are justifiable provided significant or at least noticeable increases in accuracy are obtained by the analysis.

I certainly agree with Mr. Gallot's statement that the designer needs a simplistic working tool which he can rapidly and easily use in the decision-making process. Therefore, sophisticated or detailed subroutines and parameters which add to the complexity and run time of the analysis but do not significantly alter the accuracy of the results within the design constraints that the engineer is working, need not and should not be included. On the other hand, as helicopter requirements continuously increase, there is less margin available, and the highly accurate, finely tuned analytical models such as advocated by Mr. Arcidiacono are mandatory for detailed analyses. We are rapidly approaching the state where quantum improvements are no longer possible and all are actively seeking the inclusior of those parameters which provide a two to five percent improvement in accuracy.

As evidenced by these two papers, I think both types of models can be developed and used with relative confidence provided the approach described in the papers is followed; that is, an accurate bookkeeping of the assumptions that were made in obtaining the solution and a detailed assessment of the resulting limitations of the analysis due to these assumptions. As a general rule, increasing accuracy in a math model implies increased complexity due to the inclusion of additional parameters. As the equations become more cumbersome, simplifying assumptions are made to insure mathematical solutions. These assumptions almost always limit the range of conditions for which the results are valid. Hence, detailed bookkeeping is essential lest the developer, and certainly subsequent users, attempt to use the analysis on problems for which it has limited capability or for which it was never intended.

One other point under philosophy. The high speed computer has allowed significant advancements in rotor loads analyses. It provides mathematical approximations to complex equations never before attempted. However, this dependency on computers is sometimes disturbing; therefore, I strongly endorse both author's comments concerning the requirement for a good understanding of the physical phenomena before it can be confidently included in a math model.

The last topic I wanted to touch on was those areas where the two analyses just presented were deficient, or better put — those areas in which the techniques for predicting rotor loads have to be expanded. The U. S. Army has just recently entered into a program to develop a new utility transport helicopter. Two major objectives of this typical development program are increased reliability and increased survivability. Generally increased survivability requires increased maneuverability while increased reliability requires longer component life and reduced vehicle vibrations. Accurate prediction of the rotor loads and stresses plays a vital role in achieving these objectives within prescribed design cost goals.

Figure 1 is a plot of the maneuvering envelope, typical of those desired for helicopters of the transport class. The outer boundary is the transient envelope for which the helicopter must be designed. The point at 150 knots is a 1.75 C's maneuver; it is desired that an aircraft be capable of attaining and sustaining this condition for three seconds. Unfortunately, neither of the analyses described has the capability to analyze these operating conditions. I realize that there is a semi-transient version of Mr. Arcidiacono's analysis available; however, as he acknowledges, most transient analysis available today neglect wake effects as well as control system and airframe dynamics.

Yet, if we are to achieve these design objectives of increased maneuverability and increased component life, we must be able to analyze the maneuvering flight condition and accurately calculate rotor loads and stresses. As you move up and to the right on the plot shown here, blade life deteriorates rapidly. In addition, stall flutter boundaries will be exceeded so that rotor blade loads, vibrations, and control loads increase exponentially.

Of course, it is to be expected that this type of analytical capability will introduce additional undesirable complexity as well as increase computer run time. However, it would appear that as long as these items could be kept within reasonable bounds and the validity of the model well established, one computer run which, although long, produced the desired results, would be more cost effective than fifteen short runs whose results were questionable or not applicable.

One last point for the case of increased accuracy in rotor loads prediction techniques.

Figure 2 is a plot of the trend in desired vibration levels versus speed for the cockpit and cabin of transport helicopters. It is interesting to note that the level is 0.05 G's from minus thirty knots to plus 150 knots as compared to previous requirements which were 0.15 G's. These stringent vibration requirements are due to the need for increased reliability and are based on the results of recent studies which show significant reductions in maintenance and improved reliability as a function of vibration levels.

Current concensus indicates that these objectives are attainable. However, even with vibration attenuation devices, significant detuning will be required in the fuselage. Finite element structural static stress modeling techniques properly combined with a weights and inertia model, condensed into a dynamic response model of the fuselage is within the state-of-the-art to aid in this structural detuning. However, these models can never be any more accurate or valid than the rotor loads and forces which are required for input data.

The era of "cut and try" in the development of helicopter systems is rapidly passing. As research, development, and procurement costs continue to escalate, it is imperative that analytical techniques in such areas as rotor loads be improved and utilized to build confidence into a design prior to commitment to hardware. However, one word of caution which cannot be overemphasized, regardless of how accurate the results, they are worthless unless they can be used and applied by the designer.

Within the constraints for which the analyses were intended, both the paper by Mr. Gallot and Mr. Arcidiacono represent outstanding efforts in the area of rotor load prediction capability, and I congratulate them both.

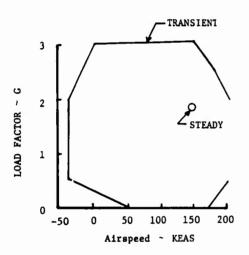


Figure 1. Typical Meneuvering Envelope for Future Helicopters.

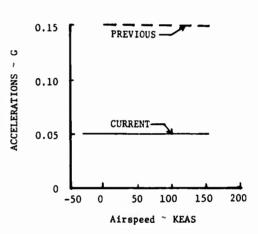


Figure 2. Trend in Desired Vibration Levels Versus Speed.

# Discussions of Paper 3 "Calcul des Charges sur Rotor d'Hélicoptère" présenté par M.Gallot

No Comments.

# Discussions of Paper 4 "Helicopter Rotor Loads Prediction" presented by P.J.Arcidiacono

R.A.Piziali: On the last slide, where it shows lifting line versus lifting surface, "Was there any representation of the shed wake of the airfoil for that or was it quasistatic?".

P.J. Arcidiacono: It is present in the lifting surface analysis as a continuous wake.

R.Loewy: What size vortex core was used in the analysis?

P.J.Arcidiacono: Bob, usually we take .1 of the chord, however, we recognize that sometimes when the vortex is close enough to the blade that assumption breaks down, such as for lifting line representation of the blade.

J.I.Bluhm: You gave computing times of one, two, and seven minutes. Are those for steady state? And, if so, I assume you've got some for the unsteady state.

P.J. Arcidiacono: That's only for a steady state operating condition. The model is for an unsteady aerodynamic case.

W.P.Jones: I'm rather surprised that so much emphasis has been placed on lifting lines in so much of this work. Certainly now lifting surface theory is available for fixed wing aircraft I would feel that that could be adapted to helicopter blades as well.

**P.J. Arcidiacono:** We have tried lifting surface techniques on hovering rotor under a recent contract with the Air Force and the report should be out on that very shortly. We found that it was not an insignificant task to apply the vortex latice method to the problem. The complexity of the program already is so large that it just takes a matter of time to get your analysis updated. Time does not always permit sitting back and updating your analysis.

**J.J.Cornish**: Could you tell me if the 6 chord length delay in the effects of stall in the blade, is primarily from maneuvering flight or hovering flight?

P.J.Arcidiacono: It was developed primarily from data on oscillating airfoils which presumably would apply to any flight condition. In other words, a lot of people have taken airfoil data from the wind tunnel at various conditions, including reduced frequency, angle of attack, Mach number, etc. If you look at that data and try and figure out where dynamic stall occurs and reduced it to a minimum number of parameters, the 6 chord lengths gets you from the inception of the stall back to the effect in the wake.

J.J.Cornish: Does that also work in coming back?

P.J. Arcidiacono: Really, there are a lot of little tricks to coming back, however, the analysis was used in predicting the response of the two-dimensional airfoil, and works fairly well.

W.J.McCroskey: I really don't understand the philosophy behind the 6 chord lengths even being there.

P.J. Arcidiacono: It's a pragmatic approach. We need something to analyze that now. We need better approaches.

W.J.McCroskey: I don't understand that even as a rational for a starting point.

P.J.Arcidiacono: I'll have to confess that I don't have the details on this since it was done at Sikorsky prior to my going there, but it does take a finite amount of time before the rotor starts to stall, and what is that time a function of? They probably just cast about looking for a way to handle this in a nonlinear manner and this was the approach that they decided upon.

# REVIEW 3 by J.L.Shipley

W.Z.Stepniewsky: We confuse our representation with mathematical models, when really, I believe that we should consider these physical mathematical models. With emphasis on the first part, physics. This means you have to make in your mind a complete picture of the physical aspects that you design your model from, from a logical point of view, then you apply the mathematics. The present analysis of the rotor loads is largely based on vortex theory. I believe that with this approach, given more time and more computer capability, will not give more accurate results. Thinking that if your theory presents pure cores of vortices or more vortices and thus will provide more accurate results is not necessarily true. You must look into the physics of this. Everything is based on a strong belief that basic theory of our loads is applicable everywhere, and this is not so because of the actual wake dynamics; however, quite fortunately, if we make a mistake about a physical structure it is not too important usually. We must realize that we only try to approximate physical realities by constructing the physical/mathematical models.

Probably the best way to start is with kinimatic theory of gusts. Attempts have been made to develop new aerodynamic theory based on gusts, but I do not know what happened to that effort. As long as we try to approximate with reality, maybe an approach similar to what Mr Piziali suggested in understanding the actual pressure distribution, this may be the closest thing to kinimatic theory of gusts, if you can go directly into interaction between particles and the surface that can generate to it. It seems like in all of our approaches to the physical/mathematical models we must keep in mind how much of this is physics and how much is mathematics, and clearly distinguish that from now on, unless we understand the physics, the mathematics will only increase the cost of the analysis without providing increased understanding of the problem. The last point I would like to make is again basic philosophy. It appears to me that in the prediction of airloads, we are trying to apply a deterministic approach to problems which are basically probabilistic in nature. Maybe it would be more honest with ourselves and get good results if we recognize this fact, and try to apply probabilistic approach from the very beginning. This, we would recognize that we know so much within these limits. Then use that basis and the data in a probabilistic approach, with a physical mathematical model, which may be more realistic than a deterministic approach.

**R.Gabel**: I would like to ask Peter Arcidiacono about some papers that Sikorsky has published on some look up tables that they have developed for aerodynamic stall. Is that being used any more?

**P.J. Arcidiacono:** The answer is yes. However, right at the moment, as John Shipley had pointed out before we can really have a complete tabulation we must have data on the various airfoils; and that in itself is a major task. This data must then be crossplotted. The results that I showed you did contain that unsteady model and in the curves on normal force and moment coefficient that data was contained in that tabulation; however, to overcome the shortcomings of that model, we've been working on the time delay model as well.

I'd like to make a comment to something that John Shipley said in his review. John, you stated that we shouldn't put anything into the analysis that we don't completely understand.

J.Shipley: Peter, I think what I was trying to say is that we should understand the physical phenomena before we try and develop the mathematical model. This follows very closely to the comment that Mr Stepniewsky has been trying to make; but I do think that the unsteady aerodynamics are significant enough and we understand them well enough that we do have to start to include them now in our analysis.

**P.J.Arcidiacono**: I would just like to point out that it's a delicate balancing act between developing new technology and determining when it should be included in our analysis. Somehow the designer has to have available to him periodic up-date and the question of when you do that I guess could be a topic of discussion all night.

M.Gallot: Mr Shipley stated that we have no knowledge of unsteady drag problems. Maybe you did not catch it when I was presenting my paper, but I talked about our model including lift drag and pitching moments have all been measured in some work that has been done by the French. I assume that these results will be published sometime. The testing included three different models. They included a 0012 airfoil, a 0012 modified with a droop snoot and a cambered airfoil. The tests will be finished in about a year. I have another comment. I think that the mathematical models that are developed ought to be tailored to the particular problems and needs that we have. For instance, if you were dealing with a rigid rotor, there are a lot of things that you might want to drop out of the analysis, and there are other things that you may not consider necessary if you were working on an articulated rotor. I think that the rigid rotor case is a bit more complex.

J.J.Cornish: I have been very interested to note that most of the comments about our problems with the unsteady aerodynamics have been made by the aerodynamicists. They themselves are then forming a selfindictment and I accept part of that. I would like to caution that there are two solutions to this problem and both of them are infinite in their scope. One of them that has been mentioned by Mr Stepniewsky that involves the basic molecular interaction gas dynamics problem, the other is of course the catalogueing of specific airfoil performances, and of course, when we add the rotor blade with the trimable tab on it we've opened up even another infinite area of data that should be catalogued. I think what is needed at this time is an overview of a middle road between an

empirical approach of catalogueing airfoils and a more fundamental aerodynamic understanding of what the problem is. Neither of these is a solution to the problem per se.

W.J.McCroskey: I think that to be a bit more optimistic on this subject of the unsteady acrodynamics that, in fact, we are making some progress and within the not too distant future we'll have a much better idea of how to incorporate all of this wealth of empirical data that now exists, and go into formulas in various ways as described by Mr Gable and also by Mr Arcidiacono. But anyway, we had a suggestion this morning that perhaps we ought to think about incorporating in our models the calculation of pressure distribution, and I for one would certainly like to discourage anyone from putting that into a helicopter loads program. I really think that the fundamental research of how the pressure distributions are derived in various unsteady cases should go on, but my feeling is that in the loads programs one ought to try to simplicate and not complicate, and that what should come out of the fundamental research on oscillating airfoils, for example, is a better knowledge of how to correct or what kind of force and moment coefficients to put in, given a first order estimate of the blade motion, and it's in this direction that I think that fundamental research is currently making some progress. I would not hold my breath until we're able to calculate theoretically the dynamic force, moment and drag coefficients on an airfoil. This work is in progress, but it's going very, very slowly and I want to reiterate what I said before. You certainly do not wish to put this extra set of calculations in your structural loads programs because it is an enormous calculation procedure, but what is coming out is a better appreciation for what unsteady blade motion parameters are the most important, so that when you then try to see what airloads are created on a blade element, to answer that question you will need two things. One is, you will need knowledge of the blade element parameters that we are in the process of defining, such as a nondimensional pitch rate, amplitude ration factors and so on. You need those as inputs to some black box that has as outputs Cm. Cd and Cn, and then the other thing you need is the construction of this black box. We should be able to tell the people who are building these loads prediction programs - what are the parameters that they ought to specify if they wish to ask questions about the force moment coefficients, and then from the data that are available we'll be able, I think in the very near future, to construct that little black box that tells you what Cm, Cn, and Cd are if you put in the right parameters, and also the answers will not be perfect, will not be absolute, but I think in an engineering sense, they're going to be very useful. And then the other thing that has been raised is the issue of having to test in an unsteady environment over all kinds of conditions every profile family that you might want to consider, and that's obviously not practical, it's not so obvious, but I firmly believe it's definitely not necessary.

P.F.Yaggy: What I was hoping is that we might get some implication from the aerodynamicist and the structural dynamicist as to what each can do to help the other. So far, I've heard mostly confessions that neither one really knows real well what they're doing, and that they're not really approaching any point of understanding very rapidly. Perhaps I am missing something, but it seems to me that we must be doing something in the very near future to gain an understanding of what these interaction problems might be in such a fashion that we may be able to concentrate on those with the abilities that we have and perhaps channel the efforts being carried out on those areas that might be mutually productive and make sure that we're not moving into the areas that are counter-productive. We're not going to get anything out of the computers until we develop a real understanding of some of these phenomena and the areas both in the aerodynamics and in the dynamics that contribute to the total problem.

R.A.Piziali: In line with what Mr Yaggy discussed, what are we trying to do? We're trying to improve our ability to predict blade stresses ultimately. What is limiting that? What are the constraints? Is it the dynamics aspects of the problem of the simulation, or is it the aerodynamics aspects? I'm not sure that it is both. I think with a little bit of effort, you might be able to do some clever subcomponent validation, if you will, of the dynamics aspects of the problem independently of the aerodynamics aspect, but no one forces something into your dynamic representation. I say that the proper experiment will allow you to find out if the dynamic representation you are using is limiting, and determine how good it is. Likewise, the aerodynamics aspect, for example, somewhat in line with the continuous wake and the skewed wake, not so much for itself, but as an example of how you can through proper comparison and validation see whether the simulations they're using are weak or not. Where are the weaknesses? When you look at the end result of the total model you can't tell. You must take it apart and validate the pieces.

ROTOR SYSTEM DESIGN AND EVALUATION USING A GENERAL PURPOSE HELICOPTER FLIGHT SIMULATION PROGRAM

bν

Richard L. Bennett, Ph.D.
Aeromechanics Engineer
Bell Helicopter Company
P.O. Box 482
Fort Worth, Texas 76101
U.S.A.

#### SUMMARY

New helicopter rotor systems are designed and existing configurations are evaluated by means of a general purpose helicopter flight simulation computer program. Discussed in this paper are both the analysis incorporated in the program and examples of the results obtained from the program. The three major parts of the analysis are: (1) mathematical model of an elastic rotor based on the modal technique, (2) rotor aerodynamics, and (3) basic rigid vehicle flight mechanics. The interrelationship among these three parts are discussed. The program has been used in support of the following phases of rotor system design and evaluation: (1) rotor blade frequency placement, (2) wind tunnel simulation, (3) steady state flight simulation, and (4) transient or maneuvering flight simulation.

#### INTRODUCTION

A family of helicopter flight simulation programs, designated C01, has been under development at Bell Helicopter Company for the past decade. The U.S. Army, through the U.S. Army Air Mobility Research and Development Laboratory (USAAMRDL), and the U.S. Air Force, through the Flight Dynamics Laboratories (FDL) of the Aeronautical Systems Division (ASD), have had an important role in increasing the capabilities of the program.

The development has followed certain guidelines. First, the analysis must describe a wide variety of helicopter configurations—single rotor, compound, tandem, or side—by—side; it must also cover a broad range of flight conditions—hover, transition, cruise, or high speed. The analysis must have an uniform texture; i.e., the level of complexity of the different phases (aerodynamic, dynamic, and rotor analysis) must be uniform. The program must be applicable to diverse types of analysis—performance, stability and control, or rotor loads. The program must be user oriented in terms of preparing the input data and interpreting the results. And finally, the output format must facilitate comparison with flight and tunnel test data. Contained in Figure 1 are some of the types of problems that can be studied with C81.

Performance

Power Required; effects of speed, density altitude, and variations of configuration parameters

V-g Capabilities
Sustained
Transient
Climbing, Diving
Stability and Control
Trim Conditions

Control positions, gradients, margins in level, climbing, diving, turning, or accelerated flight

Stability Characteristics

Root locations, frequency and damping of coupled flight modes Mode shape and analysis

Response Characteristics

Transfer function numerators of coupled equations from linear stability analysis

Time histories using fully-coupled, nonlinear equations Disturbances

Gusts-Step, ramp, sine-squared; Vertical, fore-and-aft, lateral Weapon recoil, azimuth, elevation, magnitude, duration Sinusoidal excitation of any pilot control Closed-loop symmetric pull up

Rotor Blade Loads

Fully-coupled time-variant aeroelastic analysis Beam, chord, and torsional loads, during steady or maneuver flight

Figure 1. Types of Problems that can be Studied with C81

The first major step in the development, completed in 1961 (Reference 1), was a digital program to calculate helicopter performance and rotor blade bending moments for level flight conditions. Blade aerodynamic coefficients from two-dimensional airfoil tests included compressibility, stall, and reversed flow effects. A logic network was developed to satisfy the requirements of trimmed flight by balancing the external forces and moments. The inclusion of coupling between the in-plane and out-of-plane blade deflections in the rotor dynamic analysis produced a significant improvement in the calculation of natural frequencies and forced response for rotor systems. The next major development was the addition of a rigid body fuselage with six degrees of freedom to simulate maneuvers. Definition of the airframe was extended to include center of gravity, mast length and tilt, and the sizes and locations of wings, elevator, vertical fin, and pylon fairing. The aerodynamic forces and moments from each lifting surface were treated separately in the calculation of maneuver capability and stability derivatives. Controllinkage ratios, engine-power controls, and external disturbances were added to simulate a wide variety of VTOL maneuvers (Reference 2). Under AMRDL contract (Reference 3), the math model was further expanded to encompass all basic rotorcraft configurations: single main rotor plus anti-torque tail rotor, tandem, side-by-side tilting rotor, and co-axial. The detailed aerodynamic and dynamic treatment of the second rotor, plus provisions for locating, orienting, and controlling both of the rotors, led to an all-purpose, generalized analysis. Two-, three-, and four-bladed rotors are considered for teetering, gimbaled, articulated, or rigid (hingeless) hubs. The effects of a rotor disc's gradual penetration of a shaped gust field were also evaluated during the study contract. Results of the study are presented in Reference 4 and summarized briefly in Reference 5.

The rotor dynamic model was modified under FDL Contract (Reference 6). This version of the analysis was used to study slowed-and stopped-rotor VTOL configurations. A time-variant analysis of the rigid blade flapping was added. Teetering, gimbaled, articulated, and rigid hubs with up to seven blades were modeled.

A time-variant aeroelastic rotor analysis, based on the modal technique, was incorporated into the C81 program during an Army-sponsored project, (Reference 7) providing a better analytical model for studying the loads, vibrations, and transient aeroelastic behavior of rotor systems. This paper will describe the aeroelastic rotor analysis and its relationship to other parts of the flight simulation program. The descriptions used in this paper were taken from USAAMRDL Technical Report 71-68A, Rotorcraft Flight Simulation with Aeroelastic Rotor Representation. That four-volume report not only contains a detailed description of the mathematical model, but also includes separate volumes for the User's Manual, Programmer's Manual, and a complete listing of the coding in FORTRAN.

# THE MATHEMATICAL MODEL

An aeroelastic rotor analysis must contain the following items: 1.) Accurate representation of the rotor dynamics which would include the effects of the centrifugal force field, blade twist, mass distribution, stiffness distribution, and the coupling effects between in-plane, out-of-plane, and torsion displacements of the blade; 2.) The blade element aerodynamic coefficients ( $C_1$ ,  $C_d$ , and  $C_m$ ) for stalled and unstalled; and steady state and unsteady flow; and 3.) An iterative process to determine the steady-state flight condition of the helicopter, and to define the helicopter's response to pilot control inputs, gusts, weapon fire, or other externally applied forces. The techniques used to implement these goals and the procedure used to bring them together are discussed in this paper.

## 1. Rotor Dynamics

The basic rotor dynamics are contained in the differential equations of motion for the combined in-plane, out-of-plane, and torsional deformations of a twisted nonuniform rotating rotor blade as derived by Houbolt and Brooks (Reference 8). These equations include the separation of the mass and elastic axes which produces the coupling between the linear and torsional deformations. The coupling between the in-plane and out-of-plane deformations is also represented. Although the equations describe the elastic considerations which influence the blade response, they give only passing attention to the aerodynamic and aeroelastic factors which also influence the blade response. The equations for free vibration are obtained if the aerodynamic forces are taken to be zero.

There are several techniques for solving the free vibration equations of an elastic structure. Among these are the Rayleigh-Ritz, Stodola, and the Myklestad methods (Reference 9). The solution to the free vibration equations can be represented as an orthogonal set of mode shapes, and associated with each mode shape, a corresponding natural frequency. These natural frequencies and mode shapes are essential to the solution technique used to describe the time-variant aeroelastic rotor behavior.

The Houbolt and Brooks differential equations cannot be solved in closed form for all combinations of the externally applied loads. Oette (Reference 10) shows how the separation of variables technique can be applied to the Houbolt equations. If the independent variables are assumed to be time t, and location along the blade x, then it is possible to write

$$\begin{bmatrix} Y\left(\mathbf{x},\mathbf{t}\right) \\ Z\left(\mathbf{x},\mathbf{t}\right) \\ O\left(\mathbf{x},\mathbf{t}\right) \end{bmatrix} = \sum_{n=1}^{NM} \begin{bmatrix} Y_{n}\left(\mathbf{x}\right) \\ z_{n}\left(\mathbf{x}\right) \\ O_{n}\left(\mathbf{x}\right) \end{bmatrix} \delta_{n}\left(\mathbf{t}\right)$$

where Y, Z, and  $\Theta$  are the total elastic deformation of the blade; Y<sub>n</sub>, Z<sub>n</sub>,  $\Theta$ <sub>n</sub> are the components of the n<sup>th</sup> normalized mode shape, and  $\delta$ <sub>n</sub>(t) is the participation factor associated with the n<sup>th</sup> mode shape. Y and Y<sub>n</sub> refer to the in-plane deformation, Z and Z<sub>n</sub> refer to out-of-plane deformation, and  $\Theta$ <sub>n</sub> and  $\Theta$ <sub>n</sub> refer to the torsional deformation of the blade. The total number of mode shapes included in the analysis is NM.

The basic differential equation for each mode shape is

$$\delta_{n} + 2\zeta_{n}\omega_{n}\delta_{n} + \omega_{n}^{2}\delta_{n} = \frac{F_{n}}{I_{n}}$$
 (2)

where  $\omega_n$  is the natural frequency of the n<sup>th</sup> mode of free vibration,  $\zeta_n$  is the structural damping, and  $I_n$  is the generalized inertia of the n<sup>th</sup> mode shape.  $F_n$  is the virtual work done by all of the externally applied aerodynamic forces and inertia forces associated with the nonuniform motion of the frame of reference, if these forces were to act through a virtual displacement equal to the mode shape.

Thus

$$F_{n} = \int_{0}^{R} (F_{y}Y_{n} + F_{z}Z_{n} + M_{\Theta}O_{n}) dx$$
(3)

where at a radial distance x,

 $F_{\underline{Y}}$  is the total externally-applied force in the Y direction  $F_{\underline{z}}$  is the total externally-applied force in the Z direction  $M_{\theta}$  is the total externally-applied pitching moment

Furthermore,

$$F_{y} = A_{y} + I_{y}$$

$$F_{z} = A_{z} + I_{z}$$

$$M_{\Theta} = A_{\Theta} + I_{\Theta}$$
(4)

where  $A_{\gamma}$ ,  $A_{z}$ , and  $A_{\theta}$  are the components of the aerodynamic forces; and  $I_{\gamma}$ ,  $I_{z}$ , and  $I_{\theta}$  are components of the inertia forces due to the nonuniform motion of the reference system.  $F_{n}$  is later modified to account for the externally applied mechanical forces produced by precone, flapping spring, flapping stop, and the lead-lag damper. The C81 program has been modified to solve the set of modal differential equations.

To implement the modal technique in the digital computer program, the following assumptions have been made:

- a. All blade deflections are relative to a rotating coordinate system at the top of the mast.
- b. The blade is divided into 20 equal radial segments for aerodynamic and dynamic calculations.
- c. Each of the 21 segment faces has three degrees of freedom; out-of-plane (Z), in-plane (Y), and angular orientation ( $\theta$ ) of chordline about the positive x axis.
- d. The user will supply up to six normalized mode shapes to describe  $\boldsymbol{z}_n$ ,  $\boldsymbol{y}_n$  and  $\boldsymbol{\theta}_n$  for each of the 21 segment faces for each mode shape.
- e. Linear interpolation can be used to define  $\mathbf{Z}_n$  ,  $\mathbf{Y}_n$  , and  $\theta_n$  between two adjacent faces.
  - f. The maximum number of blades per rotor is seven.
  - g. The maximum number of input mode shapes per rotor is six.
  - h. The maximum number of rotors is two.
- Any assumption made in the derivation of the mode shapes will be also applicable to the entire analysis.

The program uses the modal technique which can simulate any hub type and number of blades with the proper selection and combination of the blade mode shapes. The blade

mode shapes are first calculated for all meaningful combinations of hub boundary conditions (in-plane, out-of-plane, and torsion). The blade mode shapes will be labeled either cyclic, collective, or scissor according to the following table;

#### BOUNDARY CONDITIONS

Mode Type	Inplane	Out of Plane	Torsion
Cyclic	Cantilevered	Pinned	Cantilevered
Collective	Pinned	Cantilevered	Cantilevered
Scissor	Cantilevered	Cantilevered	Cantilevered

The in-plane boundary condition is related to whether or not the mast is free to wind up in response to the total in-plane torque. A cantilever in-plane boundary condition is equivalent to assuming the mast to be infinitely stiff in torsion, while a pinned in-plane condition is compatible with a zero stiffness mast. The mast wind-up behavior can also be described by an additional differential equation. If this additional differential equation is to be used, then all reference to the mast wind up in the mode shapes must be eliminated, i.e., all in-plane boundary conditions must be cantilever. The advantage of the mast wind-up differential equation is that any degree of in-plane fixity can be represented rather than the idealized conditions used in the definition of the mode shape.

From the sets of calculated mode shapes, certain blade modes, compatible with the hub type, are selected as input mode shapes. These are then combined by the computer program into rotor modes which are used to simulate the hub type and number of blades. The first step in selecting blade modes - collective, cyclic, or scissor - to be used in the rotor simulation is the definition of the hub boundary conditions that can exist for each blade. For the hingeless or articulated hub, each blade's behavior is independent of any other blades behavior. The out-of-plane boundary condition is cantilevered (the out-of-plane slope is zero). The in-plane boundary condition depends on the torsional stiffness of the mast. It is pinned for zero torsional stiffness and cantilevered for infinite torsional stiffness. The boundary condition for blade torsional motion about the feathering axis is cantilevered.

These sets of boundary conditions are valid for both the hingeless and the articulated hubs, because inboard of the hinge the articulated hub behaves like a hingeless hub. The difference between the articulated blade and the hingeless blade is in the stiffness distributions used to calculate the blade natural frequencies.

The hub boundary conditions for a hingeless or articulated rotor can be summarized as follows:

# BLADE BOUNDARY CONDITIONS FOR HINGELESS OR ARTICULATED HUBS

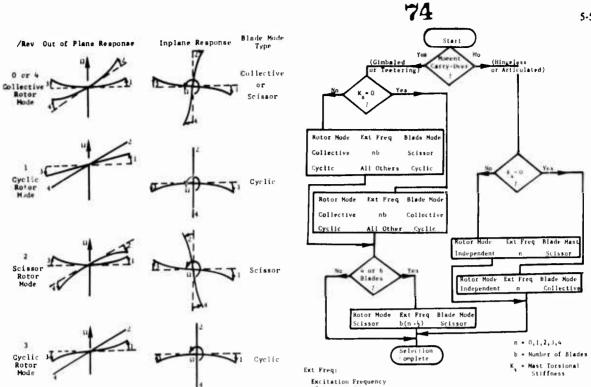
	Mast Torsional Stiffness	
Component	Zero	Non-Zero
In-Plane Out-of-Plane Torsion	Pinned Cantilever Cantilever	Cantilever Cantilever Cantilever

Recalling the previous definitions of collective, cyclic and scissor blade modes, the above table can be restated:

# BLADE MODE TYPES USED TO REPRESENT HINGELESS OR ARTICULATED HUB

Mast Torsional Stiffness	Blade Mode Type
Zero	Collective
Non-Zero	Scissor

For teetering and gimbaled hubs, which have out-of-plane moment carry-over, the selection of mode types is more difficult. The initial step is to determine boundary conditions that are compatible with the blade's response to integer-per-rev harmonic forcing functions. The four-bladed gimbaled rotor displays all possible characteristics of hubs with out-of-plane moment carry-over and will therefore be used as an example. Let it be assumed that positive out-of-plane bending (compression in top of blade) is accompanied by positive in-plane bending (tension in leading edge). Referring to Figure 2, it can be seen that for the steady or 4/rev response each blade tip moves up and aft. Thus, the out-of-plane boundary condition is cantilever. The in-plane boundary conditions depends on the torsional stiffness of the mast. If it is zero, the in-plane boundary conditions would be pinned and would require the use of collective blade modes. If it is non-zero, the scissor blade modes would be used to describe the steady and 4/rev response. The 1/rev and 3/rev are very similar. In both cases, the out-of-plane boundary condition is pinned and the in-plane is cantilevered, regardless of mast stiffness. For the 2/rev blade response, blades 1 and 3 move up and aft; blades 2 and 4 move down and forward. The out-of-plane boundary condition is cantilevered (the out-of-plane slope is zero). The in-plane boundary condition is cantilever because for the 2/rev response the total in-plane



Response of 4-Bladed Gimbaled Figure 2. Rotor to Harmonic Forcing Functions

of Rotor Mode Figure 3. Guide for Selection of Blade Mode Types to Simulate Various Hub Types

moment from all blades is zero. The resulting blade boundary conditions associated with any integer n-per-rev response for any number of blades b, can be summarized in the following table :

#### BLADE BOUNDARY CONDITIONS FOR GIMBALED ROTOR

Mast Torsional Stiffness		Harmonic		
	Component	nb	b(n-1/2)	All Other
Zero	Out-of-Plane	Cant.	Cant.	Pinned
	In-Plane	Pinned	Cant.	Cant.
	Torsion	Cant.	Cant.	Cant.
Non-Zero	Out-of-Plane	Cant.	Cant.	Pinned
	In-Plane	Cant.	Cant.	Cant.
	Torsion	Cant.	Cant.	Cant.

This can be restated in terms of blade mode shapes to describe the integer-perrev response of gimbaled rotor systems:

#### BLADE MODE TYPES USED TO DESCRIBE RESPONSE OF GIMBALTD ROTORS

Mast Torsional Stiffness	Harmonic		
	nb	b(n-1/2)	All Other
Zero Non-Zero	Collective Scissor	Scissor Scissor	Cyclic Cyclic

The column b(n-1/2) has meaning only when the gimbaled rotor system has 4 or 6 blades.

In summary, the selection of which blade modes should be used is a function of the hub type, number of blades, and analytical treatment of the mast torsional stiffness. A flow chart for selecting the proper blade modes to simulate any conventional hub is shown in Figure 3 at the top of the page.

A problem associated with the modal technique is this: The response equations, Eq (2), must apply to the total rotor system, whereas the mode shapes are calculated for one blade. To overcome this problem, a logic network has been programmed to combine the input blade modes into rotor modes, whose response is given by Eq (2). Some blade modes

can be described by one equation—for example, a collective blade mode for a gimbaled rotor where all blades go up and down together. Other blade modes are combined to form a rotor mode that requires two independent modal equations to define its position—for example, a cyclic blade mode for a gimbaled rotor which can move about two perpendicular axes (fore and aft flapping and lateral flapping).

The number of independent rotor modal equations required to describe a given blade mode is as follows: For hub types with out-of-plane moment carryover, two equations are required to describe each cyclic mode, and one equation describes each collective or scissor blade mode; for hub types without moment carryover at the hub, one independent equation must be written for each mode for each blade. Thus, the total number of independent equations required to describe any combination of hub type and number of blades depends on the blade mode type. The number of dependent equations is b times NM where there are b blades and NM input mode shapes. These dependent equations can be used to describe all hub types (hingeless, articulated, gimbaled, or teetering) for any number of blades. Each input blade mode shape must be designated by the user as to whether it is to be formed into an independent, cyclic, collective, or scissor rotor mode shape. The independent rotor modes are associated with those rotor hubs without moment carryover, and are thus able to respond at all integer multiples of the rotor speed. The cyclic, collective, and scissor rotor modes are associated with hubs with moment carryover. Collective rotor modes respond at all other harmonics. Scissor rotor modes are associated only with gimbaled hubs with 4 or 6 blades.

In Eq (4), the forcing function  $F_n$  contains the distributed inertia forces associated with the nonuniform motion of the frame of reference. These inertia forces can be obtained from the total acceleration  $a_p$  of any point on the blade. Relative to an inertial coordinate system

$$\vec{a}_{p} = \vec{R}_{H} + \vec{\rho} + \vec{\omega}_{b} \times (\vec{\omega}_{b} \times \vec{\rho}) + \vec{\omega}_{b} \times \rho + 2(\vec{\omega}_{b} \times \vec{\rho})$$
(5)

where  $\vec{R}_H$  is the linear acceleration of the origin of the moving coordinate system,  $\vec{\rho}$  is the position vector relative to a reference system attached to the top of the mast, and  $\vec{\omega}_b$  is the angular velocity of that system with components  $\mathbf{p}_b$ ,  $\mathbf{q}_b$ , and  $\Omega$ . Each mass point on the blade when subjected to the acceleration  $\vec{a}_p$  would produce a force that must be accounted for in the elastic rotor analysis. There is also an inertial moment due to the cyclic feathering acceleration that tends to twist the blade.

Each term in Eq (5) must either (1) be included in the calculation of the blade natural frequencies and mode shapes, (2) be included in the externally-applied forcing function as indicated by Eq (4), or (3) be assumed to be small enough in comparison with other terms present to be neglected.

Eq (5) can be summarized as follows by neglecting the product of any elastic blade deformation and any fuselage angular velocity;

$$\vec{a}_p = \vec{R}_H \qquad \text{(rigid fuselage linear acceleration)} \\ + \vec{X}_H + \vec{Y}_H \qquad \text{(elastic pylon acceleration)} \\ - \Omega p_b x \vec{k} \qquad \text{(gyroscopic)} \\ - \dot{q}_b x \vec{k} \qquad \text{(gyroscopic)} \\ - \dot{n} x \dot{\vec{j}} \qquad \text{(mast wind up)} \\ + \frac{2\Omega \dot{z} (z-h)}{(r^2-z^2) V^2} \dot{\vec{j}} \qquad \text{(Coriolis)}$$

where the terms without the unit vector are calculated in the fixed coordinate system and must be transformed into the rotating coordinate system. The underslinging distance is h.

In keeping with the previously introduced notation

$$I_{y} = -\left[ (\ddot{\vec{R}}_{H_{y}} + \ddot{\vec{Y}}_{H}) \cos \Psi - (\ddot{\vec{R}}_{H_{x}} + \ddot{\vec{X}}_{H}) \sin \Psi - \hat{\Omega} x + \frac{2 \Omega \dot{z} (\dot{z} - h)}{(r^{2} - z^{2}) V^{2}} \right] M$$
(7)

$$I_{z} = -(\dot{q}_{b}x - \Omega p_{b}x) M \tag{8}$$

$$I_{0} = -\theta (\rho I_{b} + \rho I_{c}) - \theta \Omega^{2} (\rho I_{b} - \rho I_{c})$$

$$(9)$$

The blade mass distribution is M; the mass moments of inertia of the blade are  $\rho I_b$  and  $\rho I_c$ . The blade azimuth position is  $\Psi$  and  $\theta$  is the blade's geometric pitch. In Eq (9), the first term on the right side is the moment produced by the cyclic acceleration of the

blade, and the second term is the centrifugal twisting moment due to the cyclic pitch. The centrifugal twisting moment due to the collective pitch is included in the blade natural frequencies and mode shapes.

The dynamic coupling between the pylon and the rotor can have a significant effect on the behavior of both systems. The effects of the pylon motion on the blade are expressed by Eq (6), in which  $\dot{\bar{x}}_H$  and  $\dot{\bar{x}}_H$  are the in-plane elastic hub accelerations. The angular velocities of the pylon (fore and aft, and lateral) are included in  $p_b$  and  $\dot{q}_b$  which appear in the gyroscopic terms in Eq (6).

The effects of the elastic blade on the pylon are modeled by two second order differential equations. The fore and aft equation can be obtained from the free body diagram shown in Figure 4. It follows that

$$I_{\mathbf{F}}\ddot{\mathbf{a}}_{\mathbf{F}} = -k_{\mathbf{F}}\mathbf{a}_{\mathbf{F}} - C_{\mathbf{F}}\dot{\mathbf{a}}_{\mathbf{F}} - PMOM + \ell_{\mathbf{F}}S_{\mathbf{F}} + \ell_{\mathbf{F}}F_{\mathbf{F}} + I_{\mathbf{F}}\dot{\mathbf{q}}$$
(10)

where  $S_F$  and PMOM are the forces and moments transmitted to the hub from the rotor.  $F_F$  represents the change in effective pylon inertia due to accelerating the blade along its radial axis.  $F_F$  is dependent on azimuth position only for the two bladed rotor systems. A similar differential equation describes the lateral pylon motion.

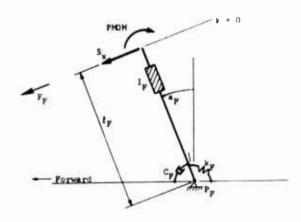


Figure 4. Model for F/A Pylon Motion

The effective cyclic pitch to the rotor is the angle between the pylon and the swashplate, and therefore any pylon motion could couple into the blade pitch. These coupling factors are input parameters for the program.

The other input parameter to the pylon equations can be obtained from a finite element dynamic model of the fuselage (NASTRAN). The principal advantage of representing the pylon motion by the differential equations is that it permits anisotropic hub restraints to be simulated readily.

Isotropic in-plane hub restraint can also be represented in the calculation of the blade mode shapes and natural frequencies rather than in the pylon equations.

The ability of the mast to twist in response to the applied in-plane torque is simulated in the C81 program by a differential equation in which it is assumed that at the bottom of mast there is an infinite torsional inertia revolving at a constant speed. Connecting this large inertia with the blade is a torsionally flexible shaft of spring rate k. The in-

flexible shaft of spring rate k. The inertia at the top of the mast is equal to the sum of the blade inertias. The forcing function for the differential equation is the applied in-plane moment from the elastic rotor. The effect of the mast wind-up acceleration on the blades is contained in Eq (6).

## 2. Rotor Aerodynamics

The blade element aerodynamic coefficients include compressibility effects and have special provisions to represent the stalled region and/or the nonsteady effects. The steady-state coefficients can be represented by analytical functions that reflect the influence of compressibility and stall. The steady-state coefficients can also be represented by airfoil data tables obtained from two-dimensional transonic tunnel tests. The user can either input the data tables for the particular airfoil, or call for an internal NACA 0012 data table that has been compiled in the program. Bivariant interpolation is used in the Mach vs angle-of-attack tables. The steady-state aerodynamics are modified to account for the effects of yawed flow over the blade segment. Elastic displacements and velocities of the blade are included in the calculation of the flow components, (Up, UT and UR), Mach number, and angle of attack at each blade segment.

An important modification to the aerodynamic simulation is the consideration of nonsteady effects. While nonsteady effects are commonly used in fixed-wing aircraft aeroelastic analysis (e.g., Scanlan and Rosenbaum, Reference 9), only the heaving (flapping velocity) terms have been consistently included in rotor aerodynamic analyses although several investigators have shown the importance of the nonsteady aerodynamics on rotarywing aircraft.

Loewy and also Timman and Van de Vooren (References 11, 12) have shown that special circulatory terms are significant in hover, but conservative results are obtained

by neglecting them (Drees, Reference 13). On the other hand, according to Carta (Reference 14) the nonsteady airloads in the stalled flow region can assume a major role in blade aeroelastic response and could produce stall flutter.

One difficulty in implementing these aerodynamic refinements is the lack of meaningful measured values over the full range of angles of attack and yaw angles applicable to the rotating blade. In particular, the wind tunnel data which are presented in terms of angle-of-attack rates ( $\dot{\alpha}$ ,  $\dot{\alpha}$ ) often consider only the rotation about the pitch-change axis ( $\dot{0}$ ,  $\dot{0}$ ) and neglect the inflow angle rates ( $\dot{\phi}$ ,  $\dot{\phi}$ ). However, the damping and inertia effects from inflow angle rates are not the same as those from pitch change rates. Simplification of the problem is unavoidable due to the lack of data. It is therefore appropriate in the calculation of nonsteady aerodynamics to use  $\dot{0}$  and  $\dot{0}$  in terms related to blade motion only, and  $\dot{\alpha}$  in terms pertaining to stall hysteresis.  $\dot{0}$  and  $\dot{0}$  include changing geometric pitch due to cyclic feathering, control motions, and elastic blade torsion.

The computer program gives the user the option of including or omitting the non-steady aerodynamic effects. The flow diagram shown in Figure 5 indicates optional paths of calculation. The application of the nonsteady effects to the calculation of incremental aerodynamic coefficients in stalled or unstalled flow is discussed in the following paragraphs.

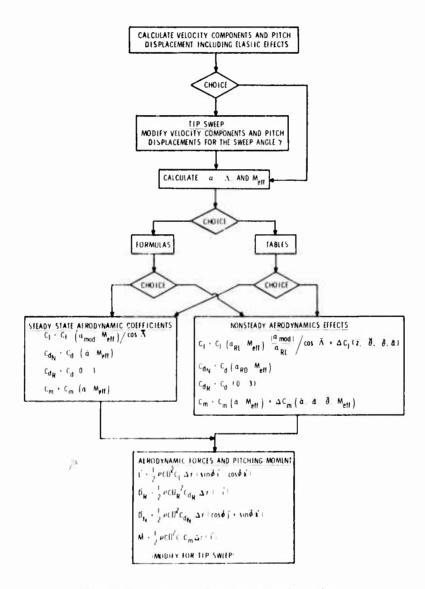


Figure 5. Flow Chart for Aerodynamics

The aerodynamic pitching moment for steady-state assumptions has first been obtained from data tables or from formulas which include the stall effects. Nonsteady effects, which include pitching velocity and acceleration of the section, both elastic and rigid body, can then be determined. The technique developed by Carta, et al (References 14 and 15) is based on data for a two-dimensional airfoil executing forced sinusoidal motion.

78

5-9

The analytical background for the theory of unsteady aerodynamics used by Carta is found in Bisplinghoff (Reference 16), and a similar discussion is given by Scanlan and Rosenbaum (Reference 9). Carta's work combines measured data and theoretical considerations to represent the nonsteady aerodynamic pitching moment.

Carta's method assumes that "the sinusoidal data could be generalized, through crossplots, to functions of  $\alpha$  the instantaneous angle of attack, an angular velocity parameter A, and angular acceleration parameter B for a given Mach number." In the considerations which follow, the parameters A and B are defined as

$$A = \left(\frac{c}{2U}\right) \dot{a} \tag{11}$$

$$B = \left(\frac{c}{2U}\right)^2 \quad () \tag{12}$$

where c is the chord length and U is the wind velocity perpendicular to the airfoil leading edge. The actual A-B values are listed in Reference 14, and are based on data from a differential pressure transducer mounted on a 2-foot chord NACA 0012 airfoil. The steady-state content of the tables is removed by requiring  $\Delta C_m = 0$  when A = B = 0. Thus, at each  $\alpha$  the original tabular value at A = B = 0 was subtracted from all entries for that  $\alpha$ . The resulting adjusted Carta tables simulate the nonsteady effect for both the stalled and unstalled regions.

The aerodynamic lift coefficient computed for steady-state can be augmented to represent nonsteady effects. The effect of the nonsteady terms on lift is defined separately for stalled and unstalled regions. The basic equation for unstalled, nonsteady lift effects is that derived by Scanlan and Rosenbaum (Reference 9). The C81 program curtails the circulation effects by assuming the lift deficiency function to be unity. The lift coefficient increment due to nonsteady aerodynamics can then be expressed as

$$\Delta C_{2} = 2\pi \left( -\frac{b\ddot{z}}{2U^{2}} - \frac{ab^{2}\ddot{0}}{2U^{2}} - (a-.5)\frac{b\dot{0}}{U} + \frac{b\dot{\alpha}}{2U} \right)$$

(13)

(14)

b is the semichord and the pitch axis (or elastic axis) is assumed to be at the 1/4 chord,

Stall hysteresis due to the variation in lift with blade pitch rate is simulated in the manner suggested by Harris (17) in which the effective angle of attack is modified  $\Delta \alpha = 61.5 \ln \left(\frac{.6}{M}\right) (\text{sign } \alpha) \left| \frac{\dot{\alpha}b}{U} \right| V$ 

The unsteady effects on the aerodynamic drag coefficient are handled in the same manner

The blade aerodynamics are usually defined in a plane perpendicular to the leading edge. However, investigators have found that the angle in the x-y plane between the wind vector and the leading edge of the blade, the yawed flow angle,  $\hat{\Lambda}$ , influences the

blade aerodynamics. From Figure 6

by modifying the effective angle of attack.

Figure 6. Effect of Tip Sweep

$$\Lambda = \tan^{-1} \left( U_{p} / U_{m} \right) \tag{15}$$

where  $\mathbf{U}_{\mathbf{R}}$  is the radial velocity component and  $\mathbf{U}_{\mathbf{T}}$ is the component perpendicular to the leading edge. The aerodynamic lift and drag coefficients reflect the yawed flow effect by modifying the angle of attack and the Mach number. Harris (Reference 17) suggests that the two-dimensional (calculated in a plane perpendicular to the leading edge) angle of attack,  $\alpha_{2-D}$  be modified to

$$\alpha_{\text{eff}} = \alpha_{2-D} \cos \Lambda$$
 (16)

Hoerner (Reference 18) emphasizes the need for replacing the two-dimensional Mach number with

$$M_{eff} = M_{3-D} (\cos k_1 \Lambda)^{k_2}$$

to improve correlation with test data. The typical value of  $\mathbf{k}_1$  is .2, and of  $\mathbf{k}_2$  is 1. The two effects are combined by

$$C_{\ell} = C_{\ell} (\alpha_{\text{eff}}, M_{\text{eff}})/\cos \Lambda$$
 (18)

The steady state drag calculation has been modified only by the includion of radial flow and the use of  $M_{\hbox{eff}}$ . According to Harris (Reference 17)

"skin friction drag force should be calculated in the direction of the resultant velocity."

This is done by computing, in addition to the conventional drag normal to the blade axis, a frictional drag along the blade based on  $U_R$ . The drag coefficient appropriate for this effect is the steady-state value based on a zero angle of attack at M=0.3.

The tip sweep angle affects the aerodynamic forces on the tip segment of the blade. For a sweep angle  $\gamma$  (as shown in Figure 6), the flow component U will influence the local inflow angle and the local angle of attack. Since the calculated drag  $D_N$  will be normal to the leading edge, the drag must be resolved into components along the perpendicular to the blade reference system axes. Thus, the component along the radial axis will not affect the elastic behavior of the blade. The offset of the aerodynamic center will cause the aerodynamic lift to produce an additional pitching moment. The tip sweep angle is added to the yawed flow angle in Eq (15).

#### 3. VEHICLE FLIGHT MECHANICS

The ability to define the trim attitude of the overall helicopter is an essential part of any helicopter flight simulation program. This trim attitude has an important effect on the aerodynamic forces acting on the rotor, fuselage, and the lifting surfaces. Trim flight can be defined as that flight condition for which the summation of the external forces  $(F_X, F_Y, F_Z)$  and moments, (L, M, N) is less than some preassigned small number  $(\epsilon)$ . An additional requirement is that the behavior of each rotor can be expressed as a periodic function of its rotational speed. This second condition can be fulfilled by defining the location of each rotor's tip path plane.

These trim conditions can be expressed as

$$|\Sigma \mathbf{L}| < \varepsilon \qquad |\Sigma \mathbf{F}_{\mathbf{X}}| < \varepsilon \qquad |\Sigma \mathbf{M}_{\mathbf{F}/\mathbf{A}}| < \varepsilon$$

$$|\Sigma \mathbf{M}| < \varepsilon \qquad |\Sigma \mathbf{F}_{\mathbf{y}}| < \varepsilon \qquad |\Sigma \mathbf{M}_{\mathbf{Lat}}| < \varepsilon$$

$$|\Sigma \mathbf{N}| < \varepsilon \qquad |\Sigma \mathbf{F}_{\mathbf{z}}| < \varepsilon \qquad |\Sigma \mathbf{M}_{\mathbf{F}/\mathbf{A}}| < \varepsilon$$

$$|\Sigma \mathbf{M}_{\mathbf{Lat}}| < \varepsilon$$

$$|\Sigma \mathbf{M}_{\mathbf{Lat}}| < \varepsilon$$

$$|\Sigma \mathbf{M}_{\mathbf{Lat}}| < \varepsilon$$
(19)

where the l/rev components of the out-of-plane hub moments are  $M_{F/A}$  and  $M_{Lat}$ . The eleven independent variables that can be used to satisfy the ten trim equations are

The l/rev components of the blade flapping angles are  $\beta_{F/A}$  and  $\beta_{Lat}$ . The flow chart of the overall technique used to trim the helicopter is shown in Figure 7. The elastic trim subroutine is shown in Figure 8.

Since there are more independent variables than there are trim equations, one of the independent variables (normally  $\Psi_{\mathbf{f}})$  must be assumed to be constant. A modification of the Newton-Raphson iterative technique is used to define the steady-state flight condition. During the trim process, the elastic rotor is treated on a quasi-static basis. That is, only the steady and the 1/rev displacement, velocity, and acceleration responses of the elastic blades are used in the calculation of the rotor airloads.

Using the eleven values of the independent trim parameters, the aerodynamic forces acting on the lifting surfaces are evaluated. The aerodynamic forces acting on the rotor disks are calculated at twenty radial positions and twelve azimuth locations. These rotor aerodynamic forces are integrated to obtain the fore-and-aft and lateral flapping moments, and the forces transmitted to the top of the mast (thrust, H force, Y force, and torque).

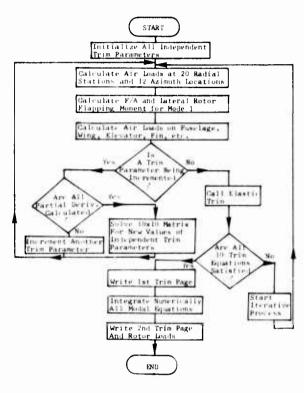


Figure 7. Trim Procedure

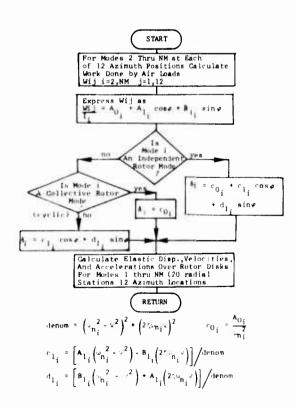


Figure 8. Elastic Trim Subroutine

The rotor forces and moments are summed with the weight components and the aerodynamic forces and moments from the lifting surfaces. If the resulting summations of forces and moments do not satisfy the trim conditions, the program uses the deviation, in conjunction with a partial derivative matrix, to predict increments to the independent variables.

These increments are then added to the previous values of the trim parameters to obtain a better approximation of the final trim condition.

If more than one mode is used in the rotor representation, the rotor aerodynamic forces are used in the elastic trim subroutine to calculate the steady and 1/rev response of rotor modes numbered 2 through NM. The flow chart for the subroutine is shown in Figure 8.

The iterative process is continued until all 10 trim conditions are satisfied. At that point, the numeric integration of the modal equation begins with the aircraft attitude and controls fixed. The four-cycle Runge-Kutta technique with a time step equal to 15 rotor azimuth is used for the numeric integra-The numeric integration continues for five full rotor revolutions, to allow the complementary solution to decay. Any rotor instability will become apparent during these five revolutions. For a stable rotor system, the solution to the modal equations must approach a periodic solution with a finite amplitude. An unstable system will have either a divergent amplitude or an amplitude with frequency component at non-integer per rev harmonics.

During the last of the five revolutions, the average rotor thrust, H force, Y force, and horsepower are calculated. These average rotor forces are then recombined with the fuselage forces calculated before the transient trim. During the last full revolution, the participation factor associated with each input blade mode is subjected to a harmonic analysis, which can be used with the bending moment distribution for each mode shape to calculate the harmonic components of the inplane, out-of-plane and torsional bending moments at each of twenty radial stations. The bending moment distributions are then restated in terms of max-min-mean-oscillatory.

Thus, the actual trim procedure consists of calculating 1) the rigid blade plus fuselage trim, and 2) the trim results from the numerical integration of the blade modal equations. The second trim process gives the initial conditions (displacement, velocity and acceleration) for each blade mode shape to be used in the transient flight simulation.

The trim procedure includes the coupling between the elastic rotor and the elastic pylon motions but neglects the rigid body fuselage motion. The differential equations which describe the fuselage response are integrated numerically in the transient-flight portion of the C81 program. The coupling between the elastic rotor and the fuselage is especially important. At each time point in the maneuver, the total hub shears and moments from the rotor blades are calculated and applied to the fuselage. These shears and moments cause an acceleration, velocity and displacement of the fuselage, which produces an inertia force distribution over the rotor

blade that, in turn, modifies the hub shear. Simultaneously, the effects of the hub velocities and fuselage velocities are included in the blade aerodynamic calculations to render a time variant aeroelastic rotor analysis. However, the forcing functions acting on the rigid body fuselage are determined from the gross rotor performance calculation, i.e., thrust, H force, and Y force.

The transient flight simulation includes provisions to define the response to pilot inputs, gust penetration, and weapons fire. The output of the maneuver simulation contains 1238 response parameters. Among these are the performance properties, fuselage motion descriptions, and the beam, chord, and torsional blade loads at 21 radial stations. All maneuver response parameters are stored on magnetic disk. The user can select any of the data to be plotted either digitally or on CALCOMP as a function of time.

The program simulates wind tunnel test conditions by balancing only the rotor flapping moments for the specific input values of mast tilt, cyclic and collective pitch. All references to the fuselage equations of motion or fuselage trim conditions have been deleted. The program is being modified to iterate on the cyclic pitch to achieve a specified flapping response for the input values of mast tilt angle, collective pitch, advance ratio, and advancing tip Mach number. All rotor forces will be in non-dimensional coefficient form to facilitate comparision between calculated and measured wind tunnel test data. The dual trim output is maintained with the second trim output giving the harmonic blade loads.

#### APPLICATION AND CORRELATION

The initial step in the use of the response program is to calculate the blade natural frequencies and their corresponding mode shapes. Typical results of this operation are shown in Figure 9, which presents the blade natural frequencies as a function of rotor speed and root collective pitch. The forcing function frequencies for the modes are also shown because the response of a mode depends on the proximity of its natural frequency to the forcing function frequencies.

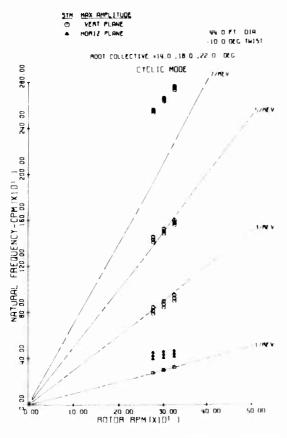


Figure 9. Coupled Rotor Natural Frequencies - T-540 for AH-1J

The primary difficulty in the calculation of the blade natural frequencies is in obtaining the required input data. This is particularly true of the torsion properties, i.e., torsional rigidity, and the locations of the shear center of the nonhomogenous blade. The torsional natural frequencies are greatly influenced by the torsional restraint of the hub. The effective stiffness of the control system can be most difficult to calculate. Another problem associated with this phase of the program is the representation of the hub impedance which can either be represented in calculating the blade natural frequencies, or be simulated in the response portion of the analysis. Representing it in the frequency calculations reduces the number of differential equations to be solved in the response portions of the program, but the frequency calculations are based on isotropic hub conditions, which may or may not represent the actual flight conditions. presenting the hub impedance in the response phase can include the effects of the hub isotropy and the coupling between the pylon motion and the blade cyclic pitch.

The C81 program is designed to be used easily in terms of preparing the input data and interpreting the output data. The input data are broken into logical groups - fuselage, main rotor, tail rotor, wing, fin, iteration logic control, flight constants, and the description of the external disturbance. All response results calculated in the maneuver portion of the program are stored internally so that at the end of the maneuver, any response item can be plotted digitally or on the CALCOMP as a function of time.

At Bell Helicopter Company, the preparation of the input data is further simplified by the use of a data library. It stores each of the logical groups for each model helicopter internally so that the user can call for the input parameters in the logical group by the use of one card. All input parameters for the UH-1D wing can be called by the term WINGUHID. The logical groups are also stored so that the total input data deck can be recalled by one card. These input features have simplified the task of both preparing and verifying the input data.

Computer simulation of a long flight can require a large amount of computing time. The Bell version of C81 is equipped with RESTART so that the long flight record can be

82

5-13

run in several sequential jobs, each of which can be started at any previously calculated time point. Each response data item is saved on magnetic tape. Then, when the next run is started, the tape is indexed to the proper time point, at which point the numeric integration resumes. One application of this is the open-loop simulation of a given flight maneuver in which it is necessary to ascertain the control inputs to simulate the maneuver.

The time variant aeroelastic rotor response program has been correlated with flight test data. Two examples show the application of the program to a two-bladed teetering rotor and a four-bladed hingeless rotor. The first example is from the UH-1D load level survey. Shown in Figure 10 are the oscillatory bending moments at blade station 6, as a function of forward speed. The spanwise distribution at 115 knots is shown in Figure 11. The calculated and measured shaft horsepower are shown in Figure 12.

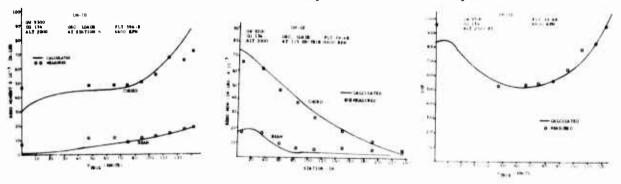


Figure 10. Oscillatory Rotor Figure 11. Spanwise Distri-Loads bution of Oscillatory Rotor Loads

Figure 12. Speed-Power Polar for UH-1D

The program has also been used to simulate transient flight conditions. The predicted and measured rotor loads at the peak g level of a heavy test helicopter are shown in the following table.

### COMPARISON OF CALCULATED AND MEASURED MAIN ROTOR LOADS

	g in	Max Osc Chord Loads	Max Osc Beam Loads
	Maneuver	Sta 5.5 (in-lb)	Sta 5.5 (in-1b)
Flight Test	1.85	362,000	9,000
C8l Simulation	1.95	365,000	10,000

The computer program was used to correlate with the flight test program of a fourbladed hingeless rotor. The stabilized level flight loads are shown in Figure 13 while the transient flight loads are presented in Figure 14.

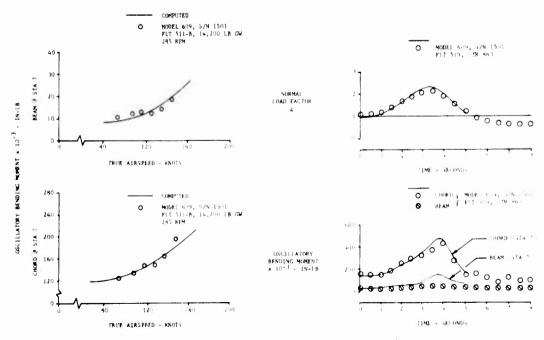


Figure 13. Correlation of Computed C81 Main Figure 14.
Rotor Beamwise and Chordwise
Yoke Bending Moments with Model
609 Flight Test, Stabilized

Level Flight

Correlation of Model 609 2.1g Symmetric Pull-Up at 130 Knots with C81 Computed Load, 12,200 Pound Gross Weight, 285 RPM

#### CONCLUSIONS AND FUTURE DEVELOPMENT

The C81 program has been shown to be a powerful analytical tool to simulate the dynamic, aerodynamic, and aeroelatic response of a helicopter, and has found its way into government agencies, industrial organizations, and academic communities. Because it is applicable to different types of studies, it has served to maintain the continuity during the design process, and is used in the conceptual, pre-design, and detail design stages. The program has been written in a logical, ordered sequence which permits changes to keep it abreast of the state of the art in VTOL technology.

USAAMRDL has awarded Bell Helicopter Company two contracts to continue the development of the C81 Flight Simulation Program. The first contract is for the use of the C81 program to predict the results of a wind tunnel test of a one-fifth-scale articulated rotor. The contract also calls for the development of correlation criteria that can be applied to rotor loads prediction programs.

The second contract concerns the aerodynamic representations for the fuselage and rotor, and the numeric integration technique used in the maneuver section of the program. The rotor aerodynamic representation will be modified to permit different airfoil data tables to be used for different blade radial segments. The fuselage aerodynamics will be enlarged so that a more refined analysis can be used for large fuselage angles of attack. A four-cycle Runge-Kutta is presently being used to integrate the fuselage and elastic rotor equations of motion numerically. A part of the second development contract is to investigate other numeric techniques (Hamming's method, predictor-corrector). Of major interest will be the numeric accuracy and computer run time associated with the proposed candidates.

The long-range development should include a technique to control the shape and natural frequencies of the blade modes as functions of the instantaneous values of rotor geometric pitch and rotor speed. Another item should be to enlarge the representations of the elastic fuselage considerations. The elastic fuselage would probably be based on the modal representation, with 24 elastic modes used to describe the fuselage.

The final item in the long range development should be an accurate, economical representation of the rotor wake and the effects of the wake on the elastic blade.

#### LIST OF REFERENCES

- Blankenship, B.L., and Harvey, K.W., A Digital Analysis for Helicopter Performance and Rotor Blade Bending Moments, Journal of the American Helicopter Society, Volume 7, No. 4, October 1962.
- Duhon, J.M., Harvey, K.W., and Blankenship, B.L., Computer Testing of Rotorcraft, Journal of the American Helicopter Society, Volume 10, Number 4, October 1965.
- USAAVLABS Contract No. DA44-177-AMC-308(T), Analytical Study of Helicopter Gust Response at High Forward Speeds, June 1965.
- Harvey, K.W., Blankenship, B.L., and Drees, J.M., Analytical Study of Helicopter Gust Response at High Forward Speed, Contract DA44-177-AMC-308(T) USAAVLAB Technical Report 69-1, September 1969.
- Drees, J.M., and Harvey, K.W., <u>Helicopter Gust Response at High Forward Speed</u>, presented at AIAA 5th Annual Meeting and Technical Display, <u>Philadelphia</u>, <u>Pennsylvania</u>, October 21, 1968.
- 6. Livingston, C.L., Bird, B.J., and McLarty, T.T., A Stability and Control Prediction Method for Helicopters and Stoppable Rotor Aircraft, Technical Report AFFDL-TR-69-123 (4 volumes), Air Force Flight Dynamics Laboratory, Air Force Systems Command, Wright-Patterson Air Force Base, Ohio, February 1970.
- Bennett, R.L., and Blankenship, B.L., Rotorcraft Flight Simulation with Aeroelastic Rotor Representation, USAAMRDL Technical Report 71-68A, U.S. Army Air Mobility Research and Development Laboratory, Fort Eustis, Virginia, Fall, 1972.
- 8. Houbolt, John C. and Brooks, George W., <u>Differential Equations of Motion for Combined Flapwise Bending</u>, Chordwise Bending, and Torsion of Twisted Nonuniform Rotor Blades, NACA Report 1346, 1958.
- 9. Scanlan, R.H., and R. Rosenbaum, <u>Aircraft Vibration and Flutter</u>, Macmillan Co., New York, N.Y., 1951.
- 10. Oette, H., Berechnung der Schlagbeige-, Schwenkbiege-und Torsionsschwingungen von Rotorblattern mit gekoppelten Eigenformen und -frequenzen, Deutsche Luft- und Raumfahrt, DLR Forschungsbericht 71-108, 1971.
- Lowey, R.G., A Two-dimensional Approximation to the Unsteady Aerodynamics of Rotary Wings, J. Aeron. Sci. 24(2), 1957.
- 12. Timman, R., and Van de Yooren, A.I., Flutter of a Helicopter Rotor Rotating in its Own Wake, J. Aeron. Sci., 1957.
- 13. Drees, J.M., Aeroelastic Rotor Phenomena and Nonsteady Rotor Aerodynamics, Annals of the New York Academy of Sciences, Volume 154, Art. 2., pp. 481-504, New York, N.Y., 1968.
- 14. Carta, F.O., Casellini, L.M., Arcidiacono, P.J., and Elman, H.L., <u>Analytical Study of Helicopter Rotor Stall Flutter</u>, Paper presented at the 26th Annual National Forum of the American Helicopter Society, Washington, D.C., 1970.
- 15. Arcidiacono, P.J., Carta, F.O., Casellini, L.M., and Elman, H.L., <u>Investigation of Helicopter Control Loads Induced by Stall Flutter</u>, USAAVLABS Technical Report 70-2, U.S. Army Air Mobility Research and Development Laboratory, Fort Eustis, Virginia, March 1970, AD869823.
- Bisplinghoff, R.L., Ashley, H., and Halfman, R.L., <u>Aeroelasticity</u>, Addison-Wesley Publishing Company, Cambridge, Massachusetts, 1955.
- 17. Harris, F.D., Tarzanin, F.J., Jr., and Fisher, Richard K. Jr., Rotor High Speed Performance, Theory vs. Test, Journal of the American Helicopter Society, Volume 15, No. 3, July 1970.
- 18. Hoerner, Sighard F., Fluid-Dynamic Drag, Published by the Author, New York, N.Y.,

# THE PREDICT ON OF LOADING ACTIONS ON HIGH SPEED SEMI-RIGID ROTOR HELICOPTERS

by

K.T. McKenzie, Cnief Technician

and

D.A.S. Howell, Assistant Chief Dynamicist
Westland Helicopters Limited, Yeovil,
Somerset, England.

#### SUMMARY

The analytical techniques employed to predict the primary loading actions of a high speed semi-rigid rotor helicopter are described. Mathematical detail is not presented as this is covered adequately by the Reference papers. The loading actions considered are overall aircraft trim balance, oscillatory rotor leading and vibratory forcing of the airframe. Some of the design considerations associated with each of these loading actions and the correlation with flight test analysis is presented. A description is given of a novel technique for the analysis of flight test results which has enabled a detailed comparison of the harmonic response of individual modes to be made. The engineering application of these techniques for the design of a high speed semi-rigid rotor helicopter has been successful and the overall correlation of the primary loading actions with flight test is good. The areas of discrepancy, defined by a detailed comparison, provide a directive for future work in the field of loading action prediction.

#### 1. INTRODUCTION

Prediction of the loading actions of a high speed helicopter is a complex task which involves a high degree of idealisation and many assumptions. When the rotor system concerned is semi-rigid the complexity of this task is amplified by a strengthening of the inter-relationships that exist between the three fundamental loading actions of overall aircraft trim balance, rotor system oscillatory loading and vibratory forcing of the airframe.

This paper will not deal in depth with the theoretical aspects of the prediction of these loading actions, but will concentrate on the use of basic design tools at an engineering level to provide a viable design solution. It will indicate an analytical technique that can be employed in the analysis of flight test results to give a definition of the actual loading actions of the helicopter in flight and will show comparisons of the predicted and actual loading actions. Such comparisons are a powerful control for the continuous process of improving the theoretical treatment in a cost effective concept.

The helicopter considered in this paper is the Westland-Aerospatiale Lynx. The Utility and Naval variants of this helicopter are shown in flight by Figure 1. This aircraft is a manoeuvrable high speed helicopter which incorporates a novel concept in semi-rigid rotor systems. At the time of writing, the Lynx holds two world speed records in its class and has successfully completed the major part of the Reverupment programme including simulated deck landing trials.

### 2. DEFIN TION OF THE BASIC DESIGN TOOLS

All the loading actions of the helicopter are dependent on the aeroelastic behaviour of the rotor system. Consequently an adequate definition of the dynamics and the aerodynamic loading of the rotor is required before any prediction of the dynamic response and the resulting loading actions can be made. The basic design tools that provide these definitions and predict the loading actions for all steady flight conditions are a series of major compatible computer programme areas. The first of these major programmes computes the natural frequencies and modes of the rotor system. These modes are then used as the generalised co-ordinates or degrees of freedom of the rotor system by a second programme that computes the response of these modes and the overall rotor performance. The loading actions for this computed flight condition are then defined by the third programme of the series which uses the loading associated with individual modes, defined by the first programme, and the response of these modes calculated by the second programme.

## 2.1. Prediction of the Natural Frequencies and Coupled Modes of the Rotor System

The natural frequencies and orthogonal modes of the rotor, coupled in flatwise and chordwise bending and torsion, are calculated using an extension of the Holzer - Myklestad method which is described in detail by laskson and Eisley (Ref. 1). This method involves a mathematical idealization which consists of dividing the blade into a number of spanwise segments having uniform elastic properties along their length which simulate the actual flexibilities of the blade. Between each of these segments there is considered to be a twist discontinuity to simulate blade twist and any psuedo - static torsional deformation due to torsional effects such as propellor moment. The local mass and local inertia of the blade are represented by a series of discrete mass points and associated three dimensional inertias which are considered to be situated between the uniform elastic segments. The mass is distributed so that both the local and total spanwise centre of gravity of the blade is maintained. The sum of the inertias



FIG. 1. WESTLAND-AEROSPATIALE LYNX - UTILITY AND NAVAL VARIANTS

normal to the blade spanwise axis represent the torsional inertia of the blade. The local spanwise inertias at the mass points primarily allow for the centrifugal force coupling effects due to the spanwise distribution of blade mass. The local mass and inertia, blade segment flexibility and the twist discontinuity are mathematically represented by a series of mass, elastic and twist matrices, the elements of which are defined by Ref. 1. Starting at the tip of the blade, the mathematical idealisation therefore consists of a mass matrix, an elastic matrix and a twist matrix followed by another mass matrix and so on until a final matrix is defined, either mass or elastic, at the blade root. Each of these matrices when multiplied by a column matrix of the values of tending moments, shears, slopes, deflections and torque at a particular radial station defines the changes that occur in these parameters due to the local mass, flexibility or twist.

Successive multiplication of these matrices produces a linear relationship between the tip and root values of the parameters in the column matrix. Recognition of the fact that tip moments, shears and torques are zero and that certain root boundary conditions are zero enables a determinant to be formed, the elements of which are polynomials in terms of the natural frequency squared, and for a frequency equal to a natural frequency this determinant is equal to zero. Thus the natural frequencies of the rotor can be found by substituting trial values of frequency and calculating the numerical values of the determinant.

Substitution of a natural frequency into the elements of the ix produces a numerical relationship between tip and root values of moments, shears, slope, deflections and torques. As stated, boundary conditions define some of these parameters to zero, and the remaining unknown tip values can be determined with respect to an assigned un numerical relationship. Consequently, successive multiplication from atrices that provide the mathematical definition of the blade by the parameters will yield a radial distribution of moments, shears, slowed deflections and torque for the mode associated with the substituted natural frequency.

tip deflection by this ip to root of the olumn matrix of these

Definition of the moments and shears associated with each of the moss is essential for the prediction of the loading actions of the helicopter.

#### 2.2. Prediction of Rotor Performance and Modal Response

The serod, mamic loading and the response of the orthogonal modes of the rotor system, together with the rotor performance, are calculated by a method describe detail by Wilkinson (Ref. 2).

**87** 6-3

The rotor performance and aerodynamic loading calculations are based on a rotor model where the blade is represented aerodynamically by a lifting line, the lift, drag and pitching moment of which are obtained from modified coefficients interpolated from tables of two-dimensional wind tunnel data stored as functions of Mach number and angle of incidence. The modification of the coefficients introduces the major effects of sweep of the aerodynamic velocity vector due to forward speed. Deflections and velocities due to blade deformation are defined by the response of the series of orthogonal modes calculated for the rotor system by the method described in Section 2.1. There is an option on the rotor wake model employed, the wake being represented by either a simple Glauert model or the possibly more representative Vortex model.

The method consists essentially of computing the rotor performance and azimuthal history of blade deflections for successive revolutions of the rotor until convergence is obtained. The downwash is defined for a series of azimuthal positions around the disc and, after calculating the local forces acting on the blade, radial integration is performed at each of these positions in succession to define the total generalised forces for the modes of the rotor system. It should be noted that these force calculations include Coriolis forces, la, damper moments and aerodynamic damping in torsion. Having defined the generalised forces at a particular azimuthal position, temporal integration yields the modal displacement at the next azimuth position, ready for calculation of the local forces.

Calculation of the modal displacements is based on the Lagrangian modal equations of motion where each mode in turn is considered to be a generalised co-ordinate of the rotor system. Due to the definition of the kinetic energy terms in these calculations loading action predictions are restricted to steady flight conditions; so that the kinetic energy terms due to aircraft velocities are constant and thus disappear when differentiated prior to substitution into the Lagrangian equations.

The Coriolis force terms have to be calculated in the performance programme since they are non-linear being dependent on both blade velocities and deflections.

The generalised forces associated with the Lagrangian modal equations of motion are obtained by differentiation of the work function with respect to each mode in turn, where the work function is defined as the work done by the forces acting on the blade for an incremental displacement of each mode.

From the rotor performance and azimuthal history of blade deflections thus calculated, the loading actions associated with the main rotor system can be derived.

#### 2.3. Prediction of the Loading Actions

Fourier analysis of the modal response calculated in the performance programme, together with the definition of the moments and shears associated with each mode, gives the basic information required for definition of the loading actions.

From the summation of the first harmonic flap moments and in-plane shears associated with each mode, the steady trim moments and forces in the fixed co-ordinates of the aircraft can be calculated and a computed trimmed flight condition obtained.

Summation of the bending moments associated with the response of all modes at a particular azimuthal position will yield a radial bending moment distribution in the flatwise and chordwise directions. Computation of these bending moment distributions around the azimuth will produce the calculated phased oscillatory fatigue loading for the rotor head and blade. The harmonic content of this loading  $c_{ij}$  be obtained from the Fourier analysis of the modal response by the summation of the Fourier co-efficients of bending moments of all the modes for each harmonic in turn.

Prediction of the vibratory forcing of the airframe can be made from the appropriate harmonic rotor centre-line 1000 of moment and shear force. For example, for a four blade helicopter such as the Lynx, 1000 cell and co-ordinate fourth harmonic pitch and roll moments are the resultant non-rotating moment vectors of the third and fifth harmonic flap moment vectors in the rotating co-ordinates of the rotor system. Longitudinal and lateral fourth harmonic shears are similarly formed from third and fifth harmonic in-plane shears. The fourth harmonic vertical shears and oscillatory rotor shalt torques however are the sum of the fourth harmonic components for each blade, since their reference axis is the same in both fixed and rotating co-ordinates.

Consequently from these calculations a prediction of the primary loading actions can be made for the helicopter in steady flight conditions, including quasi-steady maneuvre, from which a considerable degree of understanding of loading in free manoeuvring conditions can be obtained. The major load environment of the rotor system is dominated by the complex of total loading and moment system of the aircraft and for a semi-rigid rotor with its balance between strength and flexibility in its aeroelastic behaviour, it is of vital importance to control the balance.

## 3. AIRCRAFT TRIM PREDICTION AND CORRELATION WITH FLIGHT

It is essential, at the conceptual stage of a design, to calculate carefully the predicted longitudinal trim conditions for the aircraft through its flight envelope and loading/C.G. ranges. Only thus can a considered judgement be made concerning the avoidance of potentially damaging conditions and the consequential margins for manoeuvre. This can be even more essential in the case of the "semi-rigid" than in the case of the small off-set rotors, as there is little rotor disc tilt contribution to the aircraft moment and the flap-bending strength of the "semi-rigid" elements has to be balanced with rotor dynamic and aircraft stability considerations.

#### 160 KNOTS ISA / SL

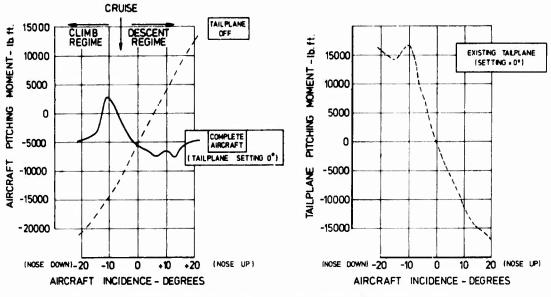
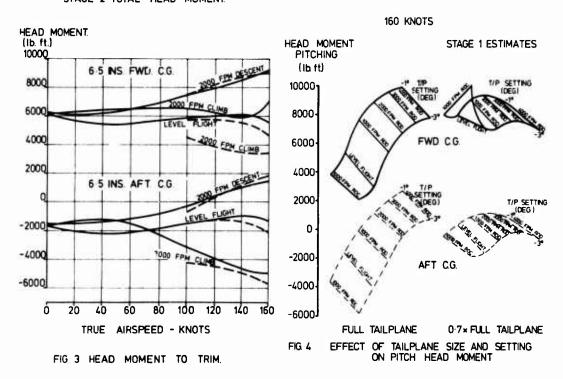


FIG. 2 PITCHING MOMENT CHARACTERISTICS FROM WIND TUNNEL DATA.



With the critical flight envelope requirements and aircraft size dictated by the aircraft specification and the loading/C.G. ranges governed by the operational tasks, the designer has to satisfy the ensuing trim conditions by manipulation of shaft tilt, tailplane size/setting and rotor head strength. Considerations of dynamic stability will contribute to the ultimate solution and care has to be exercised to prevent such considerations jeopardising the optimisation of rotor head strength.

It was decided at an early point to adopt a two-stage approach to the rotor head strength/trim knowledge. STAGE 1 was to comprise a simplified longitudinal trim only approach involving the use of derivatives computed from the coupled mode rotor performance programme described in Section 2,

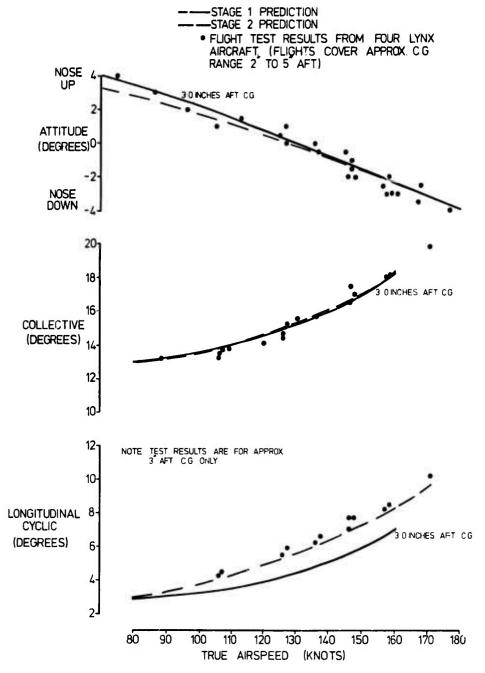


FIG 5 COMPARISON OF PREDICTED LEVEL FLIGHT TRIM WITH FLIGHT TEST.

together with facilities for quickly assessing the effect of tailplane size/aspect ratio and setting; tailplane moment being represented by a linearisation of the characteristic shown on Fig. 2, with data factors being available to multiply this moment by any number, to represent a tailplane size/aspect ratio change and to shift the curve by a given amount to represent a setting change. STAGE 2 also used the coupled modes derivatives, but in this case an equivalent hinge offset and spring stiffness were computed and used in a conventional six-degrees of freedom model. After optimisation of the correlation of the derivatives with those of the coupled modes programme this model was used for the trim calculations around the ultimate aircraft configuration.

The flight conditions considered must all lie within the quasi-linear operating regime of the main rotor and STAGE 1 adopted an approach based on rotor forces and moments generated by cyclic and collective pitch settings in association with the flight path vector.

For a given airspeed and atmospheric condition linear derivatives were computed from several runs of the performance programme, e.g. holding shaft angle, longitudinal and lateral cyclic angles constant, two or three levels of collective pitch were run to yield thrust, pitching moment, rolling moment and flapping angle collective pitch derivatives. The derivatives for the other parameters were similarly obtained, together with assumed constants for thrust, pitching moment, rolling moment and flapping angle evaluated from averaged results from the programme runs.

- 2500 FRM ROD
- LEVEL FLIGHT
- 1500 FPM ROC RESULT-FLIGHT TEST AT 2.7° FWD CG

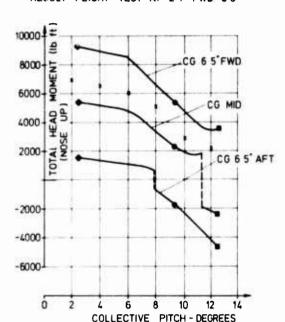


FIG 6 COMPARISON OF HEAD MOMENT IN CLIMB AND DESCENT AT 140 KNOTS WITH FLIGHT TEST.



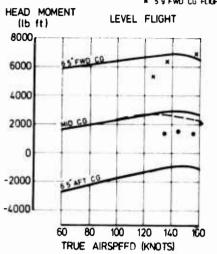


FIG 7 COMPARISON OF HEAD MOMENT IN LEVEL FLIGHT WITH FLIGHT TEST

To simplify and speed up the solution the complete six degrees of freedom system was reduced in that side force and yawing moment equations were deleted and the rolling moment equation was admitted with the left hand side set to zero. The most significant cross coupling effects on pitch were

A further approximation was made in the force normal to the shaft, which was represented by a vector obtained by resolution of the thrust vector through the flapping angle and an assumed constant evaluated from the averaged results from the programme runs.

Associating these equations with the aircraft force and moment equations, defined by airframe characteristics from wind tunnel work, shown by Fig. 2, leads to trim values of shaft angle (aircraft attitude), collective pitch, longitudinal cyclic, a first approximation to lateral cyclic, flapping angle, and rotor head pitching moment.

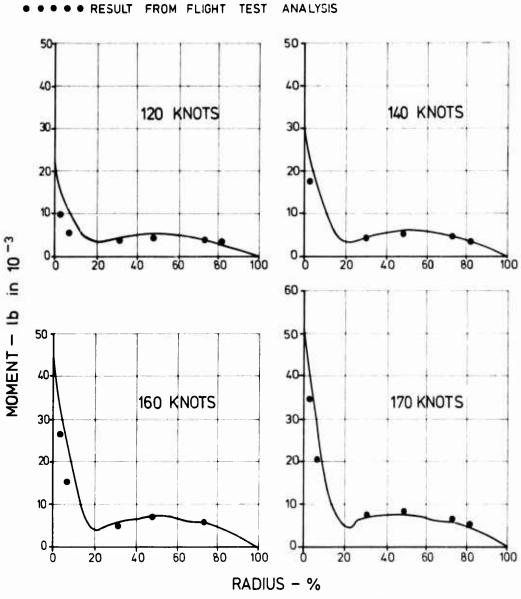
Fig. 3 shows the calculated rotor head pitching moment to trim over a range of airspeed and rate of climb/descent for the predicted required C.G. range of the Lynx, for both STAGES 1 and 2. This chart gives data for the tailplane configuration and setting eventually selected. Fig. 4 shows the effect of tailplane size and setting as given by STAGE 1. The critical condition for trim of high speed descent/forward C.G. is clearly shown, the asymmetry of the situation being caused by the built in shaft tilt of 40.

The trim predictions and flight test results have correlated well for a high speed helicopter having a rotor of novel and original design together with a significant tailplane contribution. The correlation of pitch attitude, collective pitch and longitudinal cyclic is shown by Fig. 5. In general the agreement is good, the major differences being found in the case of STAGE 1 longitudinal cyclic angles which are about 1° less than measured values. The important parameter of head moment to trim in climb, descent and level flight is shown in Figures 6 and 7 which indicate a reasonable degree of correlation with flight results. Thus both STAGES of the trim solution, which rely implicitly on the validity of the derivatives obtained from the performance programme, show that in this respect the programme is useful and reliable.

# 4. FATIGUE LOADING OF THE ROTOR SYSTEM

Fatigue loading prediction is a vital aspect of semi-rigid rotor design philosophy and forms a corner-stone of the design process. The flexible elements and blades of the rotor must satisfy certain stiffness requirements and give an adequate fatigue life under the loads experienced throughout the flight spectrum. In the lag plane the stiffness constraints are defined by fundamental lag frequency and damping for ground and air resonance stability and in the flap plane by rotor control power and flight stability considerations. These stiffness requirements are not rigid, but they do form additional constraints of the rotor design process, together with fatigue loading, endurance limits for the choice of materials for the rotor system, and acceptable rotor weight and inertia which, together with stiffness, influence frequency spacing and hence vibration.

The fatigue loading of the rotor is dependent on a number of factors one of which, aircraft trim throughout the flight envelope, has been covered in Section 3. Other factors are the distribution and magnitude of the oscillatory aerodynamic loading, the shape of various modes, the



CALCULATED

Fig. 8 FLAP/FLATWISE OSCILLATORY MOMENT.

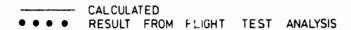
bending moments associated with each of these modes and the frequency spacing of the natural frequencies of these modes with respect to harmonics of rotor speed. It is this interaction with other considerations which make it imperative that the prediction of fatigue loading forms an integral part of the iterative design process. The computer programmes described in Section 2 provide the basic tools for this interrelated design process.

The calculated loading for level flight conditions obtained from these programmes provide basic design information which has to be interpreted and related to an overall spectrum of loading. After consideration of the various roles of the aircraft, it was decided that high speed level flight, with the possible exception of some extreme C.G. positions, with extreme manoeuvres, should produce stress levels equal to or less than the endurance stress limits of the rotor materials. The nett fatigue life of the rotor would then be defined by fatigue damage done during manoeuvres, transient and excessively turbulent conditions.

The flexible element cross-sections are determined by flexibility requirements in the flap and lag planes and the stress limitation for the section imposed by the design stress level under design loading. The design stress level is obtained from suitably factored fatigue test data and the design loads from suitably factored calculated loading. This nominal loading must then be related to a design loading spectrum to give a fatigue life basis for the rotor system.

#### 4.1. Predicted Fatigue Loading

The design loading for the Lynx predicted in this manner has produced fatigue lives equal to, or in excess of the design fatigue lives of the major rotor components, although discrepancy between observed and calculated loading in the lag plane is an area of concern. As indicated the design loading incorporates certain factors based on an engineering interpretation of the



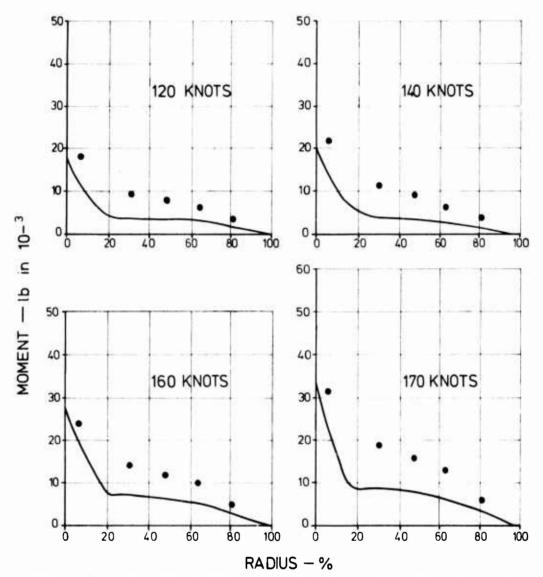


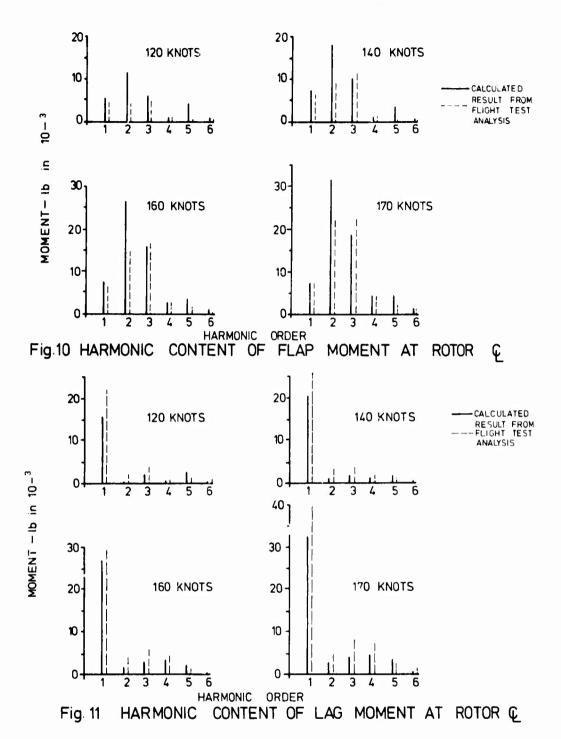
Fig. 9 LAG/CHORDWISE OSCILLATORY MOMENT.

calculated loading for steady flight conditions; therefore comparison of the recorded flight loads with the design loading would not provide a true indication of the theoretical load prediction capability. Consequently a range of level flight cases have been computed specifically for comparison with a particular set of flight test records. These cases are level flight conditions at 120, 140, 160 and 170 knots airspeed for an aircraft weight and C.G. of 8650 lb. and 1.26 in. aft.

The calculated radial distributions of flap/flatwise oscillatory bending moments for these four speeds are compared with the recorded moments by Fig. 8. The agreement obtained for the flatwise bending moment along the blade for this speed range is good in each of the four cases. There is also relatively good agreement for the flap bending moment of the rotor head flexible elements, an agreement which improves with increased airspeed.

The lag/chordwise oscillatory bending moment radial distributions are compared with the recorded moments by Fig. 9. From this comparison it is evident that the calculated moments are less than those recorded in flight, significantly less in the case of the chordwise moments for the blade.

In both planes the moments are basically the vectorial sum of a series of harmonic moments and therefore the harmonic content of the calculated and recorded moments should be compared in an assessment of theoretical prediction capability. Such a comparison has been made and is illustrated at two stations which are the rotor centre line for the rotor head flap and lag moments, and 31% radius for the blade flatwise and chordwise moments. These comparisons are made by the bar charts of Figures 10 and 11 for the rotor centre line, and Figures 12 and 13 for 31% radius.



The first harmonic flap bending moment for the rotor centre line is in agreement with the recorded moments due to the simulation of the flight trim conditions. There is however, some discrepancy in the first harmonic moment calculated for the blade station, in particular for the 120 knot case. The largest discrepancy between calculated and recorded loads is for the second harmonic, especially for the hub where this component is calculated to be twice the order of the recorded component, whereas for the blade the second harmonic is calculated to be less than that recorded. The calculated third harmonic moment is slightly lower for the blade than the recorded component. The fourth and sixth harmonics are small components and the calculations predict orders comparable to those recorded. The calculated fifth harmonic moment is consistently greater and does not exhibit the same trend as the recorded component. Higher harmonics are not shown since their magnitudes are insignificant at these stations.

In the lag/chordwise sense the pre-ominant component is the first harmonic. The calculated magnitude of this component of the la, moment at the rotor centre line is comparable with that recorded in flight. For the chordwise moment on the blade however the calculated value is approximately a factor of two less than that recorded. The calculated values of the second, third and fourth harmonic moments are also significantly less than those recorded for both the hub and blade.

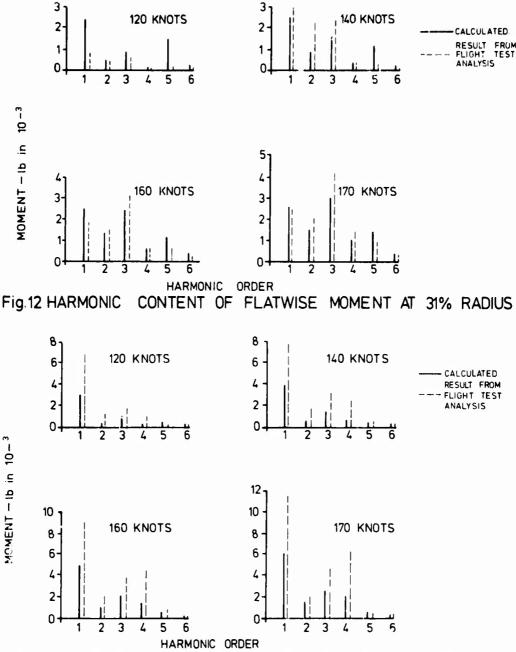


Fig. 13 HARMONIC CONTENT OF CHORDWISE MOMENT AT 31% RADIUS

#### 5. VIBRATORY FORCING OF THE AIRFRAME

Vibratory forcing prediction is an important aspect of helicopter design, but as yet it does not form a corner-stone of the rotor design philosophy in the same way as performance, fatigue life and ground/air resonance considerations. This is because concern about subjective and environmental vibration is not a primary issue in the same way as aircraft performance, strength and freedom from catastrophic instabilities. Vibration is an important factor however and due consideration must be given to this problem at the rotor design stage. The most obvious area of influence is the frequency spacing of modes with respect to the harmonics that are the source of the fixed co-ordinate vibratory forcing. This influence is evident in the mass and stiffness distributions of the blade which, by virtue of their effect on the modal response of the rotor system, have an influence on the basic issues of rotor design. Consequently prediction of vibratory forcing must remain an integral part of the rotor design process.

It should be noted that attenuation of the vibratory forcing by careful rotor design will enhance the fatigue life of components such as the main rotor shaft and gearbox in addition to reducing the vibration levels which will enhance crew comfort and ability, and equipment endurance.

The source of the vibratory forcing in terms of harmonics of rotor head moments and shear forces is known and has been indicated by Section 2.3. The modal contributions to these harmonic components and the relative importance of the various modes are not generally known. A novel technique has therefore been developed for the analysis of flight test records which defines these modal contributions

and attempts to realise more of the information potentially available in these records.

This technique has been termed the "Modal Analysis Technique" and is fundamentally the solution of a series of simultaneous equations. Each of these equations is formed by making the assumption that the measured bending moments at any instant in time, at any radial station, are simply the vector sum of the moments associated with the response of all the modes of the rotor system. From the definition of the bending moments associated with a unit response of the modes, one side of an equation can be written in terms of the moment for each mode at the particular radial station and the unknown response of each of these modes. This side can then be equated to the measured bending moment at that radial station. The measured bending moment is obviously some function of time and consequently the response of the modes must also be functions of time. In practice the measured bending moments are basically harmonic in content and can therefore be represented by a series of Fourier coefficients. It follows that the response of the modes can also be represented by a series of Fourier coefficients and the equation indicated above can be written for each of these coefficients in turn. Thus at every radial station where tending moments are measured, a set of equations can be obtained in terms of the Fourier coefficients of the unknown modal response and the measured bending moment. From the equations for each of these radial stations, a set of simultaneous equations can be written for a particular harmonic coefficient. It is on these sets of equations that the modal analysis technique is based and their solution will yield the Fourier coefficients of the unknown modal response.

This definition of the modal response allows the accurate interpolation and extrapolation of phased harmonics of bending moment from the rotor centre line to the blade tip. From the shear force distributions associated with individual modes the shear force distribution can simultaneously be obtained. The rotor centre line values of moments and shears obtained in this manner enables the fixed co-ordinate vibratory forcing to be defined and phased in terms of pitch, roll and yaw moments; vertical, longitudinal and lateral shear forces.

For the Lynx which has a four blade main rotor the predominant vibratory forcing is at the fourth harmonic, with other components of diminishing order at the eighth harmonic, twelfth harmonic and so on; together with residual components of other harmonics that are not integer multiples of the number of blades. In this paper discussion will be concentrated on the predominant fourth harmonic component.

ROTATING BLADE COORDINATE CONVENTION.
FLATWISE BENDING - POSITIVE UP
CHORDWISE BENDING - POSITIVE FORWARD
SECTION TWIST - POSITIVE NOSE UP.
DIRECTION OF ROTATION - POSITIVE ANTICLOCKWISE
FROM ABOVE.

FIRST HARMONIC OF ROTOR SPEED DENOTED BY  ${\mathfrak N}$ 

FIXED COORDINATE CONVENTION.

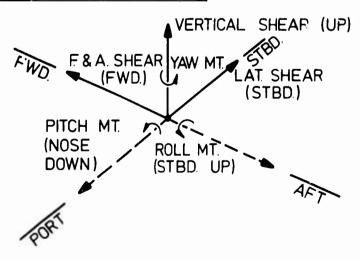
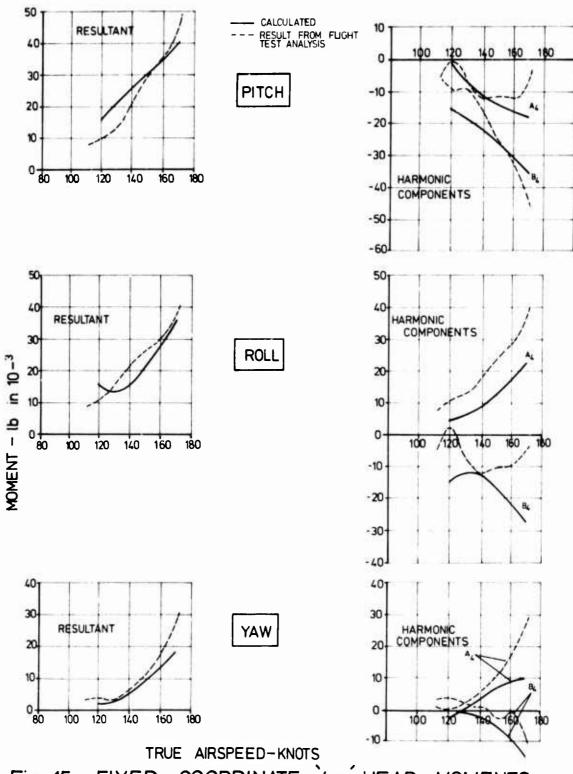


FIG. 14. FIXED AND ROTATING COORDINATE SIGN CONVENTION.



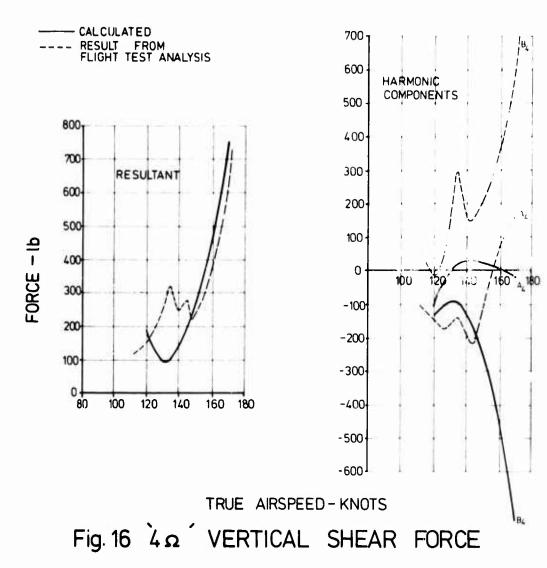
# Fig. 15 FIXED COORDINATE 42 HEAD MOMENTS

The sign convention used in this section for both fixed and rotating co-ordinate moments and forces is defined by Fig. 14, and is the result of adopting the sign convention used by Isakson and Eisley (Ref. 1).

# 5.1. Comparison of Predicted and Derived Vibratory Forcing

The range of computed level flight cases used in Section 4, together with the modal analysis of the flight test results, will be used for this comparison of vibratory forcing.

The resultant vibratory moment vectors in the pitch, roll and yaw directions for the speed range considered are shown by Fig. 15 which also shows the harmonic coefficients of these moments. From these graphs it can be seen that for the resultant pitch and roll moment vectors



there is good correlation between the predicted and derived values. The harmonic coefficients of the pitch moment also show a reasonable degree of correlation, both in magnitude and phase. With the roll moment, despite the good correlation of the resultant vector, there is some difference in both magnitude and phase of the harmonic coefficients. The calculated A4 coefficient, although low by an approximately constant amount, exhibits the same trend as that derived from flight test whilst the predicted B4 coefficient does not exhibit the same trend, and is consistently greater than that derived from flight test. It can also be seen that there is good correlation between the predicted and derived values for the resultant yaw moment and reasonable correlation of the harmonic coefficients of this moment, apart from a phase shift, up to a speed of 160 knots. Above this speed the calculated A4 coefficient does not exhibit the steep increase of the derived coefficient.

The predicted and derived vertical shear force resultant vectors are shown by Fig. 16, together with the harmonic coefficients of these forces. From this figure it can be seen that the very steep increase of the resultant vertical shear force above 150 knots is predicted although the relatively small increase that occurs between 120 and 130 knots is not predicted. Although the trends of the calculated harmonic coefficients show some degree of correlation they are of the opposite sign to the derived coefficients.

The resultant vibratory forces in the longitudinal and lateral directions are shown by Fig. 17 together with the harmonic coefficients of these forces. From these figures it can be seen that there is no correlation in the longitudinal direction. In the lateral direction there is some degree of correlation between the calculated and derived resultant and the A4 coefficient, although the B4 coefficient exhibits no correlation.

# 5.2. Harmonic and Modal Contributions to the Pitch and Roll Moments

It is apparent from the preceding comparison of vibratory forcing that the pitch and roll moments are the most predominant forcing components and will therefore need to be considered in greater detail. These moments are the result of the third and fifth harmonic bending moments in rotating co-ordinates; a comparison of the predicted and derived rotor centre line values of these moments is made by Fig. 18. This comparison shows a reasonable agreement between prediction and flight test for B3, the major coefficient. The trend exhibited by the A3 component is not

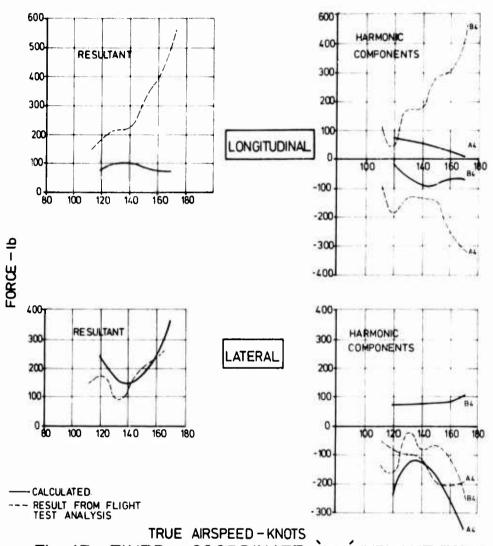
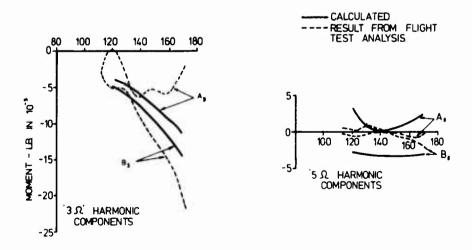


Fig. 17 FIXED COORDINATE 42 INPLANE FORCES



TRUE AIRSPEED - KNOTS

FIG 18 ROTOR C 3  $\Omega$  AND 5  $\Omega$  FLAP MOMENT PER BLADE

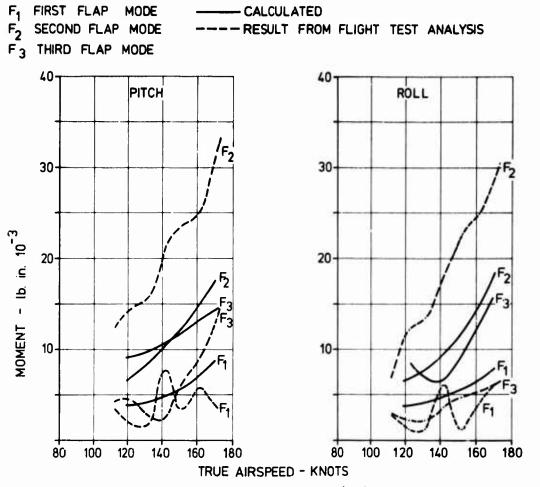


FIG. 19 MODAL CONTRIBUTIONS TO FIXED COORDINATE 42 PITCH AND ROLL MOMENTS.

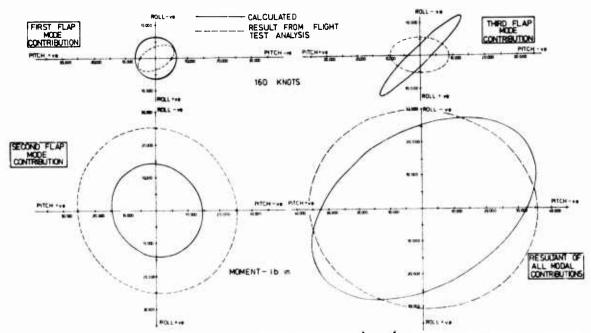


Fig 20 POLAR PLOT OF FIXED COORDINATE '4 Q' HEAD MOMENT VECTOR

predicted by the calculations which give a generally higher value for this coefficient. The trends and magnitude of the fifth harmonic components do not agree with the flight-test results, the calculated moment being two to four times the measured moment. Consequently the predicted pitch and roll moments have a greater fifth harmonic content than those derived from flight-test, but since the fifth harmonic is small compared with the third, this in itself is relatively insignificant.

These harmonic moments and consequently the fixed co-ordinate pitch and roll moments are due to the forced response of the modes of the rotor system. The predicted modal contributions to the pitch and roll moments are therefore compared with those derived from the modal analysis of flight test results. The comparison is shown by Fig. 19 for both the pitch moment and the roll moment. From the analysis of flight-test results it is apparent that the major contribution is from the third harmonic response of the second flap mode, with smaller contributions from the first and third flap modes. There are contributions from the other modes which have been omitted for clarity since they are all less than those shown. Although the second flap mode contribution is the largest of the predicted modal contributions it is significantly less than that derived from flight-test analysis. Whereas the contribution from the third flap mode is significantly greater than the derived contribution. The predicted and derived contributions of the first flap mode are of the same order.

The modal contributions to the pitch and roll moments can be clearly seen from a polar plot of the fixed co-ordinate head moment vector. These can only be plotted for a particular flight condition and an example of such a plot is shown by Fig. 20. This compares predicted and derived total resultant moments and modal contributions to these moments for 160 knots level flight, the maximum design cruise speed of the Lynx. It is evident from this polar plot that the predicted and derived moment vector loci for both the resultant and modal contributions are significantly different. The derived second flap mode moment vector loci is approximately circular and can be seen to contribute the major part of both the pitch and roll moment vectors, whereas the predicted vector loci although also approximately circular is only some 56% of the derived vector. The predicted moment vector loci for the third flap mode exhibits a phase shift and is a more acute ellipse than that derived from flight-test although the magnitudes are more comparable. It can be seen that the magnitudes of the contributions from the first flap mode are comparable although the predicted loci in this case is more circular than that derived from flight-test. Comparison of the resultant moment vector loci shows their magnitude to be comparable although the predicted loci is more elliptical in form.

#### 6. COMPARISON OF THE MODAL HARMONIC RESPONSE

The aeroelastic behaviour of the semi-rigid rotor, manifest by the response of the modes of the rotor system, defines all the loading actions of the helicopter associated with the rotor. A detailed examination of the harmonic response of the modes will therefore identify the harmonics and modes that account for the observed discrepancies. The significant harmonic modal amplitudes for the first three flap modes and the first two lag modes are compared with the predicted amplitudes by Figures 21 to 25. These figures show the harmonic response of these modes in terms of the principal tip deflection of the mode. From Fig. 21 it can be seen that the predicted response of the first flap mode agrees quite well with the derived values apart from the fourth harmonic, however this is relatively small compared with the first and second harmonics. Local irregularities in the trends of these harmonic amplitudes with increasing speed are not generally predicted. The predicted harmonic

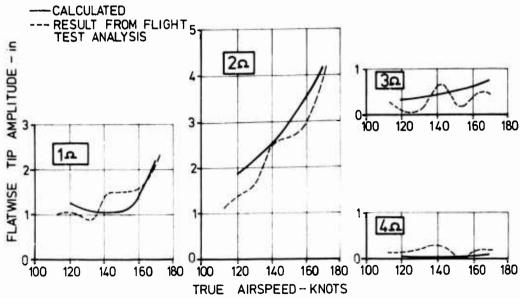


Fig. 21 FIRST FLAP MODE HARMONIC AMPLITUDES

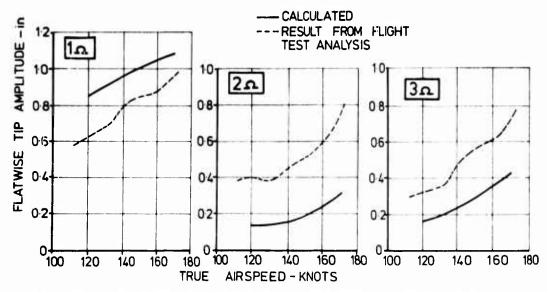


Fig. 22 SECOND FLAP MODE HARMONIC AMPLITUDES

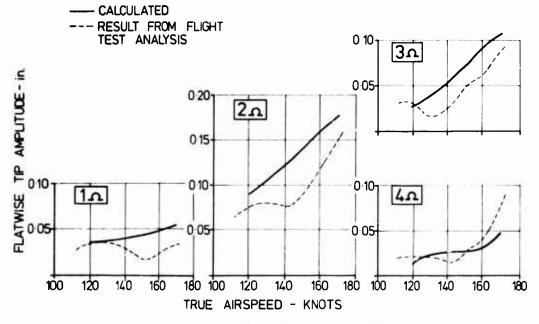


Fig. 23 THIRD FLAP MODE HARMONIC AMPLITUDES

amplitudes of the second flap mode whilst exhibiting similar trends with speed as those derived from flight records do not have comparable magnitudes as shown by Fig. 22. The first harmonic amplitude is over estimated, whilst the second and third harmonic amplitudes are calculated to be half the order of the derived amplitudes. The predicted trends of the third flap mode shown by Fig. 23 also in general agreed with the derived trends, with the first, second and third harmonic amplitudes overestimated whilst the fourth harmonic amplitude is underestimated above 150 knots. From Fig. 24 it can be seen that apart from the first harmonic the predicted response of the first lag mode is consistently low, although the predicted trend of the second harmonic amplitude is similar to the derived trend. The predicted harmonic amplitudes of the second lag mode shown by Fig. 25 indicate a reasonable degree of agreement for the first, third and fourth harmonics whilst the second harmonic is overestimated by a factor of approximately two.

# 7. CONCLUSIONS AND OBSERVATIONS DRAWN FROM THE COMPARISONS

In the preceding sections of this paper the three fundamental loading actions of aircraft trim, oscillatory fatigue loading of the rotor, and vibratory loading of the airframe observed in flight have been compared with the theoretical predictions. The degree of correlation of the harmonic and modal components of these loading actions has also been presented in some depth. The theoretical and analytical techniques employed for the predictions have been indicated together with the fact that in practice certain engineering factors have been applied to allow for areas of weakness in the

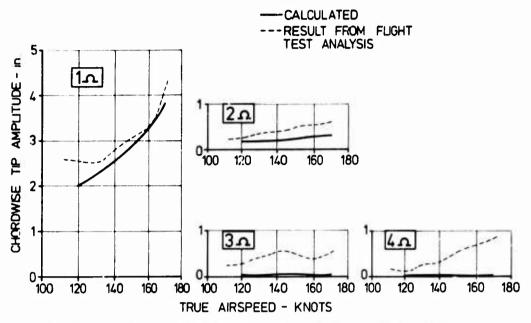


Fig. 24 FIRST LAG MODE HARMONIC AMPLITUDES

--- CALCULATED
--- RESULT FROM FLIGHT
TEST ANALYSIS

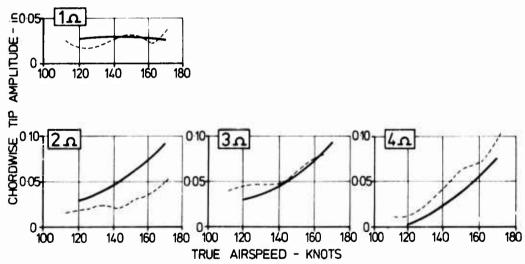


Fig. 25 SECOND LAG MODE HARMONIC AMPLITUDES

theoretical predictions. The predictions made for the Lynx have demonstrated that these design tools, accepting an 'engineering' interpretation of their results, provide a viable design capability. Furthermore the detailed analysis of flight records has been shown to provide a basis for determination of the absolute standard of the theoretical predictions and hence a directive for improving the theory. Some further observations of the discrepancies between the predicted and actual loading actions will illustrate this point.

From the examination of the harmonic amplitudes of the modal response certain factors are apparent. For instance the over estimation of the second harmonic moment for the rotor head, and the under estimation of this moment for the blade is due primarily to the fact that the predicted second harmonic response of the second flap mode, in particular the A2 coefficient, is too small. The agreement of the predicted fixed co-ordinate pitch and roll moments is not due to a good agreement of the modal contributions, ref. Section 5, but due mainly to the fact that the third harmonic response of the third flap mode is overestimated, whilst the response of the second flap mode is undor estimated. The difference in sign of the predicted B4 coefficient of vertical shear is primarily due to the B4 coefficient of the third flap mode being of the opposite sign to that derived from flight records.

Such a comparison in detail of predicted and derived modal response therefore defines the mechanisms that account for the observed discrepancies in the loading actions. From this information it is apparent that relatively small differences in the magnitude and phase of the response of individual modes can produce a significant nett discrepancy. Thus in the light of this knowledge the theory and computing processes used can be assessed with the objective of improving the predicted amplitudes of the most significant modal responses.

Since the response of any mode is determined by the 'work function' - the product of that mode and the aerodynamic loading - the definition of the mode shape, the aerodynamic loading distribution, and the integration technique employed are critical parameters. Modification of the definition of the tip less effects, increasing the number of radial stations at which the above product is defined and substantiation of the actual mode shapes are areas which offer the potential of improving the predictions. There are inevitably errors in the magnitude and distribution of the aerodynamic loading, in particular the oscillatory loading, due to a lack of data for oscillating aerofoil characteristics and the effects of sweep of the velocity vector due to forward speed on these characteristics. The definition of downwash, even the vortex ring model, is a simplified model of the actual downwash and this may have a significant effect on the harmonic response of the higher order modes. Similarly interference effects with the airframe, tail rotor and vorticies shed from the rotor hub may influence the response of these modes.

The theoretical techniques employed have demonstrated both their viability as design tools and their potential to give a detailed definition of the rotor's aeroelastic behaviour in steady flight conditions. With the present standard of these techniques however there are some significant areas of discrepancy. Their successful application for the design of the Lynx, and the good degree of correlation obtained with flight test for the overall prediction of loading actions, should not detract from the effort required to understand the reasons for the discrepancies, an understanding that will inevitably improve the detailed correlation. It is only from a position of such an understanding that the helicopter designer can achieve an enhanced degree of control over all the loading actions at the design stage of the aircraft.

## REFERENCES

- 1. G. Isakson and J.G. Eisley.
  - Natural Frequencies in Coupled Bending and Torsion of Twisted Rotating and Non-rotating Blades. NASA CR-65, July 1964
- 2. R. Wilkinson
  - A Description of a Computer Programme to Predict the Performance of a Helicopter Rotor in Steady Forward Flight.
  - Westland Helicopters Research Paper No. 397, May 1971.

COMMENTS ON LOADS PREDICTION
Paper by Dr. R. L. Bennett and
the paper by K. T. McKenzie and D. A. S. Howell

Henry R. Velkoff Ph. D.
U. S. Army Aviation Research
and Development Laboratory
Ames Research Center
Moffett Field, California 94035
USA

#### 1. Bennett

Dr. Bennett's paper presents the results of an extensive effort to produce a computer program for helicopter rotor loads, and for determining helicopter flight characteristics. The C-81 program offers an approach to calculate the effects of certain maneuver conditions and as such is a definite contribution. The program has been continually expanded to handle a wide variety of rotor configurations and aerodynamic factors that can influence the lift production of the blades.

Although the paper does not explicitly indicate it, the inflow modal used is based on momentum considerations with a triangular shape across the disk. It would seem that the next logical extension to the C-81 program would be to include a more complete aerodynamic representation for the rotor wake for steady flight conditions.

The program considers an extensive range of non-steady aerodynamic effects on the local airfoil sections. It includes provisions for maneuvers; however, some concern exists as to the nature of the rotor flow during the maneuver. It is not clear whether during a maneuver the wake remains unperturbed during the pilot input, develops instantly or follows with some delay. If, as it appears to be, the wake in the program is assumed to remain instantaneously fixed, then it would seem as a logical next step that a wake model with a suitable lag could be incorporated(1).

The correlations of beam and chord bending appear to be good. It would have been interesting to see the computer program applied to other cases and other types of rotors. Time histories of calculated and measured pitching moments would be beneficial. It is hoped that in subsequent papers, Dr. Bennett can present such added correlations and will be able to continually update the program to include improved wake models as they become available.

#### 2. McKenzie and Howell

The approach presented in the paper by McKenzie and Howell is similar in principle to the methods presented by other authors in these proceedings. A lumped parameter analysis is used to define the blade modal characteristics, and a modal analysis is used to represent the rotor. The aerodynamic model used is not made specifically evident, but a Glauert model seems indicated. Of particular significance is the technique indicated whereby the modal contributions to the harmonic content of the rotor head exciting forces are presented. This represents a positive step for the designer in that it provides a direct indication as to the sources of the vibratory inputs which may be causing an undesirable level of fuselage vibration.

In addition the response of the blades in their various modes for the various harmonics is also derived from the flight test data. Thus it is possible to get a better estimate of the validity of the analysis with regard to the blade modal response to various harmonic inputs. The reductions of flight data to the harmonic content and the modal content is considered to be of considerable value from an intuitive and design standpoint. It would appear to be beneficial to relate the harmonic magnitudes in terms of generalized forcing functions, and these results compared with the technique presented by the Cornell Aeronautical Laboratory and indicated in Reichert's paper contained in these proceedings. Since from a broad sense both rotors are similar in having a hingeless hub configuration, such a comparision of generalized forcing functions between the two sets of data could prove revealing and fruitful.

The significance of the paper would be enhanced by an inclusion and discussion of pitching mements.

#### 3. Evaluation of Modal Analysis

A study of the papers presented in this meeting reveals that with perhaps one exception all the methods utilize a modal approach to handle the rotor in aircraft trim and response and in the blade moment analysis. In several of the papers, greater and greater attention is being given to fine details of the rotor loads problem. The availability of high speed computers allows this to be done with such relative ease, that one may tend to overlook possible questions on the mathematical foundation of the methods used. It may be possible that the very technique of analysis could introduce changes of the same order of magnitude as the fine physical refinements being incorporated.

The concern arises due to the use made of generalized Fourier techniques in modal blade analysis. If one writes out the differential equation of bending of the blade, includes the "aerodynamic forcing functions" and attempts to obtain separation of the spatial and time terms, then it becomes evident that such separation cannot be achieved. If one considers blade motion in a vacuum, separation will occur. The blade modes so obtained form a valid set of functions which can be used to describe blade bending,

Thus although the modal techniques using the "in vacuum modes" can be used to produce given blade deflections, they may not truly be solutions to the partial differential equations of motion for the blade. For usual engineering accuracy the technique may still be adequate. For highly refined computer analyses, second order effects such as this could possibly be significant.

## REFERENCES

1. Crews, S. T. Model Tests on Unsteady Rotor Wake Effects, AIAA Journal of Aircraft, Vol. VI. Hohenensser, K. H.

Commantaire préparé
concernant les communications (5 et 6 )
par R. HIRSCH
Ingénieur Docteur - Aérospatiale D.E.P.

Le programme de calcul objet de la communication présentée par M.R.L. BENNETT de la Bell Helicopter Cy, est remarquable par l'étendue des problèmes qu'il traite, lesquels couvrent pratiquement l'ensemble des questions soulevées par la mécanique du vol des hélicoptères (performances - stabilité et manoeuvres - vibrations - effets des rafales, etc...). Ainsi permet-il une étude très efficace des projets de réalisation auxquels les ingénieurs sont confrontés, et sur ce plan beaucoup de problèmes apparaissent résolus.

M. Mc Kenzie développe les résultats d'application des méthodes de prévision appliquées par Westland au si intéressant appareil qu'est le Lynx; en ce sens son exposé prend un caractére concret particulièrement attachant, qui indique en général de bonnes corrélations entre prévision et expérience.

Toutefois, si l'on se place sur le plan de la connaissance même des phénomènes, en particulier d'ordre aérodynamique, dont sont le théâtre les rotors d'hélicopteres, certains de leurs aspects doivent, nous semble-t-il, être soulignés.

Les premières questions que l'on aimerait voir précisées par l'un et l'autre auteurs sont les suivantes :

A quelle modélisation du sillage tourbillonnaire et à quelle méthode de calcul des effets d'induction de celui-ci a-t-on recours ?

S'agit-il d'un réseau de tourbillons enveloppant des nappes hélicoTdales ou d'un calcul de vitesse induite moyenne ?

Le calcul des distributions d'efforts est-il effectué à partir du principe de la surface portante, ou de la ligne portante ? Si oui, a-t-on tenu compte des intéractions entre profils ?

Par ailleurs, les données aérodynamiques entrant dans le programme sont apparemment issues des propriétés des profils d'ailes fonctionnant en écoulement bidimensionnel (même si des corrections empiriques sont éventuellement mises en oeuvre.)

D'après nos propres confrontations entre théorie et mesures, cette manière de considérer les choses est valable dans les cas où le coefficient local de portance n'excède pas la valeur du décrochage bidimensionnel, et où aucun phénomère transsonique n'est en cause ou sur le point de se manifester.

Ceci rejoint les conclusions que l'on peut tirer de l'exploitation d'un rapport NACA ancien (NACA.RM. L8A3Oa) concernant l'expérimentation d'ailes élancées en flèche, en régime compressible.

Nous examinerons ses points d'un peu plus près, ci-après par calcul-expérience ayant trait, l'une aux conditions de décrechage sur une pale d'hélice, donc <u>stationnaires mais en régime tourmant</u>, l'autre sur l'influence des effets transsoniques, ici instationnaires, sur les pales d'un rotor expérimental d'hélicoptère.

Auparavant il semble que la question de l'introduction des conditions instationnaires dans la formation des circulations et celle des efforts aérodynamiques sur les pales demande un examen particulier.

Nous sommes en plein accord avec M. BENNET pour constater que les expérimentateurs nous ont fourni une documentation importante sur le seul cas de rotation instationnaire des <u>profils</u>. (bidimensionnel).

Rien ou presque, ne nous est donné sur le pilonnement et le tamisage, et absolument rien sur le cas d'oscillation du courant extérieur, pourtant d'importance primordiale, ici. Enfin l'influence des effets tridimensionnels et celle du dérapage demeurent très peu explorées. Nous en sommes réduits à des développements exclusivement théoriques, dont on sait la fragilité, s'ils ne sont pas corroborés et redressés pas à pas par une expérimentation adéquate, c'est-à-dire portant sur les mécanismes mêmes des phénomènes (et non sur des comparaisons globales).

Si nous avons bien compris l'exposé de M. BENNETT, son programme prend en compte les effets instationnaires par incorporation dans l'évaluation des efforts des termes vitesse et accélération de déplacement et rotation des profils, auxquels on associe des coefficients d'influence liés à la fréquence réduite.

Cette manière de faire, très adaptée aux problèmes de flutter entretenu, possède ici à nos yeux, le petit inconvénient d'éliminer du problème des termes purement transitoires. Ils y figurent naturellement si les paramètres influents sont introduits dans le calcul par des intégrales de Duhamel.

En résumé, si les programmes présentés par MM. BENNETT et Mc KENZIE répondent remarquablement aux besoins des études concrètes, on ne doit pas considérer que tout le problème de la connaissance de la prévision des caractéristiques de fonctionnement des rotors d'hélicoptères est encore réellement résolu, notamment en ce qui concerne :

- l'évaluation complète des effets instationnaires autres que ceux résultant de la rotation de profils
- celle des effets de compressibilité non subsoniques et celle des effets de "décrochage" en régime tridimensionnel dérapé.

Il serait à nos yeux important d'orienter les recherches pour essayer de combler en profondeur ces graves lacunes.

Telle peut être notre conclusion.

#### ANNEXE 1

On trouvera ci-dessous un diagramme relatif à la distribution radiale du ressaut de pression apparaissant à la traversée d'un disque d'une hélice travaillant, à calage de pas de pale constant, pour des paramètres d'avancement  $f = \pi \Lambda = V_n D$  décroissants, de sorte que la pale entre en décrochage progressivement.

La prévision de calcul fondée sur l'hypothèse d'écoulements bidimensionnels eût donné une configuration du type indiqué en traits interrompus, toute différente de celle observée.

Cette dernière suggère la formation progressive d'une configuration tourbillonnaire analogue à celle en cornets des ailes à forte flèche sous forte incidence (déportance en extrémité).

Le coefficient de portance max. bidimensionnel était de l'ordre de 1,4. La courbe = 0,7 conduit à une valeur de ce coefficient devant dépasser 2 dans le domaine de rayons relatifs 0,3 à 0,5.

Il y a donc lieu de ne pas seulement prendre en considération les altérations des conditions de décrochage apportées par l'état bidimensionnel <u>instationnaire</u>.

Un phénomène tridimensionnel stationnaire est sous jacent, qui doit être évalué aussi.

#### ANNEXE 2

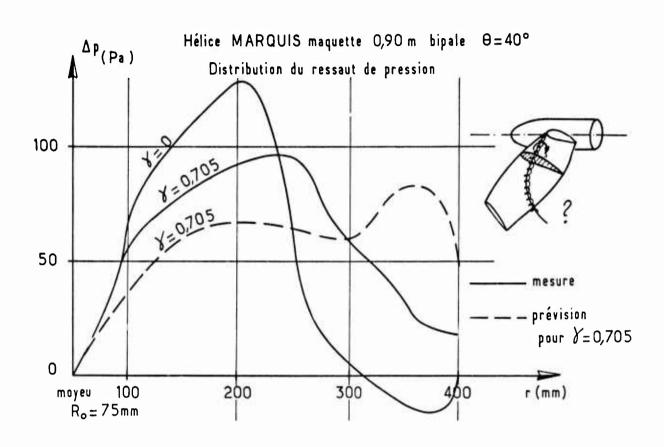
Une deuxième série de diagrammes concerne les distributions de charges aérodynamiques calculées et mesurées sur quatre profils de pales d'un rotor d'hélicoptère expérimental, pour des paramètres d'avancement croissants jusqu'à 0,6.

Nos calculs incorporaient :

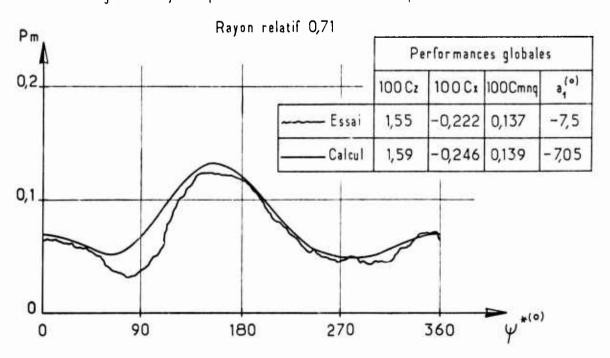
- les effets d'induction d'un système tourbillonnaire en nappes hélicoidales d'un type analogue à celui de <u>Pizziali</u>, mais aplaties et déformées pour figurer la distorsion du pas et la contraction du sillage.
  - l'introduction du pas cyclique
- la détermination des battements verticaux et horizontaux et l'introduction des composantes de vitesses qui leur sont attachées.
- les effets instationnaires touchant le développement de la circulation et celui des champs de pression (donc des efforts)
- les effets d'intéractions tridimensionnelles entre profils de pales qu'il faut absolument prendre en compte particulier si l'on a recours au schéma de la ligne portante.
- les effets de compressibilité subsonique et ceux des délais de transmission des signaux d'induction des éléments tourbillonnaires liés et libres, en création ou disparition du fait du caractère instationnaire du problème.
- les non linéarités des lois de portance et moment des profils à l'égard d'incidences variant de 180° à + 180° dans le cercle d'inversion d'après des données pour le moment seulement bidimensionnelles.

La comparaison entre courbes issues des calculs et celles issues des mesures, complétées par les courbes d'évolution du Mach et du dérapage montre bien que la prévision théorique est satisfaisante partout ou le Mach local est inférieur à la limite d'apparition des phénomènes transsoniques. Il y a désaccord local lorsque ceux-ci se manifestent.

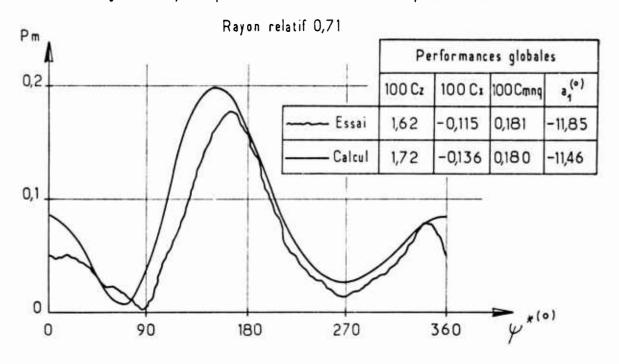
Le dernier diagramme, sur lequel pour quatre régimes de fonctionnement sont portées en grandeur et position, les résultantes d'effort sur le rotor, calculées et mesurées, indique enfin une prévision excellente du calcul, tant que ces effets soniques interviennent eu (ici pour les paramètres d'avancement inférieurs à 0,6).



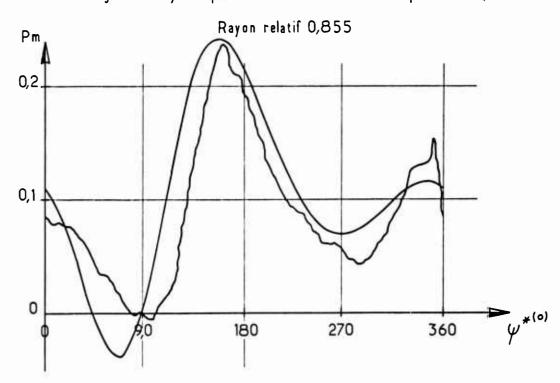
Charges aérodynamiques locales Pm-dans le tour pour  $\Lambda = 0.30$ 



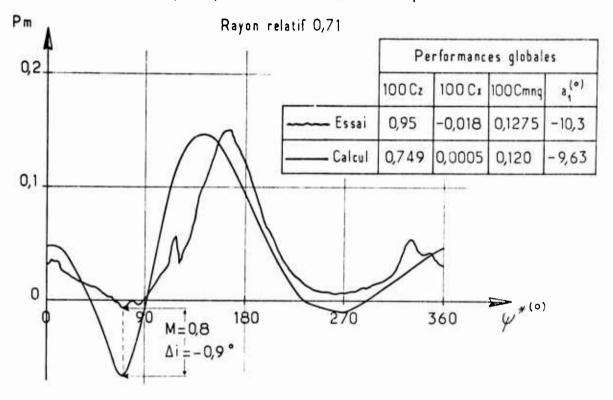
Charges aérodynamiques locales Pm-dans le tour pour  $\Lambda = 0.45$ 



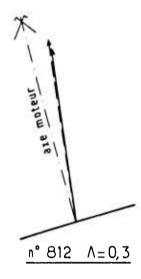
Charges aérodynamiques locales Pm-dans le tour pour  $\Lambda = 0.45$ 



Charges aérodynamiques locales Pm dans le tour pour  $\Lambda = 0.60$ 



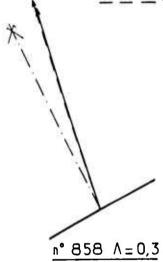
Position des résultantes aérodynamiques





\_\_\_ Essai(corrigé de l'effet de moyeu)

\_\_ \_ Calcul





#### Discussion of Paper 5

"Rotor System Design and Evaluation Using a General Purpose Helicopter Flight Simulation Program" presented by R.L.Bennett

R.Gabel: Can you illustrate any transient load correlations?

**R.L.Bennett**: In the paper that I have prepared, I have shown some transient load correlation that we have achieved on a four bladed hingeless rotor in a symetric pull up. It was for a 2g manoeuvre entered from a dive. The correlation was adequate. So I'll talk to you later and show you the figure in the paper.

**A.W.Kerr:** You mentioned there were two problems about the change in natural frequency of the blade modes with pitch and with r.p.m. Have you any capability in the programme to allow, during a dynamic manoeuvre, for the variation of r.p.m. and its effect on mode shape or frequency?

**R.L.Bennett:** Yes, internal to the programme, we have a differential equation that describes the mast wind up, which is the oscillation about the mean r.p.m., so that in the 3 by 3 table, a 2 way interpolation was used to get the effect of r.p.m. and geometric pitch.

J.L.McCloud: The previous authors showed a pretty large spread in the experimental oscillatories. Would you concur with this? Is this your experience also?

R.L.Bennett: In the oscillatory what? Pitch link loads?

J.L.McCloud: No, beam bending for example.

**R.L. Bennett:** Yes, substantial scatter has been observed in experimental data and this is one of the problems in the correlation efforts. If the manoeuvre were repeatable in all of its many parameters, I think the scatter would be reduced.

J.L.McCloud: So you believe it's the flight condition. Do you have more success, then, if you are comparing with data obtained from a wind tunnel simulation?

**R.L.Bennett:** We have recently concluded two modifications to the programme in the area of wind tunnel simulation. The first is to iterate on cyclic pitch to achieve a moment balance so that we can run the programme like the wind tunnel condition itself. We are now engaged in a contract with USAAMRDL to check out the programme against a Sikorsky wind tunnel test.

Previous results used, compared in the literature to AV LABS Report 68-3 were fairly good.

#### Discussion of Paper 6

"The Prediction of Loading Actions on High Speed Semi-Rigid Rotor Helicopters" presented by D.A.S.Howell

R.Gabel: Can you comment on calculated and measured torsional modes and pitch link loads? I did not see that in the slides.

D.A.S.Howell: The torsional modes and control loads are fairly close to what would be anticipated for a torsionally stiff articulated rotor system. At the beginning of this presentation I mentioned that we attempted to minimise torsional couplings at the design stage; we have a high degree of "matched stiffness" outboard of the feathering hinge. I think because of this design objective we do not have the torsional couplings and fairly high control loads normally associated with semi-rigid rotor systems.

J.L.McCloud: Does the Lynx present to the pilot any information about hub moments?

D.A.S.Howell: No there is no information presented to the pilot concerning hub moments. We have found that the relatively low roll inertia of the Lynx enables the aircraft to execute a roll manoeuvre without exceeding the design loading distri' ution by a significant margin. There is also a fairly novel device called the "collective — 'g' compensator", which is incorporated to improve longitudinal stability during pitch manoeuvres. This device changes collective pitch by an increment proportional to normal 'g' levels up to a limit of approximately 2.5 g. It follows that the predominant first harmonic inplane loads generated during nose-up pitch manoeuvres are reduced by this device attenuating the peak 'g' levels experienced during the manoeuvre.

113

## REVIEW 4 by H.Velkoff Reply by R.L.Bennett

First, I share your concern about the wake analysis. In C-81 a modification of the momentum theory is used to calculate the inflow. A wake analysis is certainly something to be desired especially in the transition regime. I apologize for not presenting any of the data that I have as a function of time instead of only in terms of oscillatories.

The aerodynamics in the manoeuver, that you talked about, is that we are using a numerical integration on both the fuselage equations of motion and the rotor equation of motion. As the helicopter goes through the air, we, at each time step, evaluate all of the aerodynamic and inertia loads that are acting on the fuselage, wing, elevator, fin and on the rotor itself at that particular azimuth condition. I share your concern with the verification of the yawed flow model. I would only like to explain that my concern is on the verification of the non-steady aerodynamics.

I tried to indicate in my presentation that one of the major problems associated with the C-81 is the verification of any addition to it. It is structured in terms of building blocks which we test with a great amount of care. But, it is still a tremendous job to come up with a critical experiment to check out the modifications.

You were mentioning about a standardized set of data for everyone to use, is certainly an interesting concept.

I may have mentioned before, that right now, Bell Helicopter has a contract to use the C-81 programme to predict the load that has been measured by Sikorsky Aircraft on a four bladed articulated rotor-model rotor. I am sure you know that, at Bell Helicopter, we are not as experienced in articulated rotors as we are with two bladed rotors — but we are seeking to use this programme in an honest correlation effort to find out exactly where we are in terms of development of the programme.

#### LOADS PREDICTION METHODS FOR HINGELESS ROTOR HELICOPTERS

by

#### G. Reichert

## Messerschmitt-Bölkow-Blohm GmbH Ottobrunn, Germany

#### SUMMARY

A hingeless rotor system allows the transfer of large moments from the rotor to the fuselage, which results in a substantial improvement of the stability and control characteristics of the whole helicopter. This is, together with the mechanical simplicity the main advantage of the rigid rotor. On the other hand, the transfer of large moments through the hub results in different structural requirements compared to articulated rotors.

The special loading condition of the hingeless rotor helicopter will be discussed. For the prediction of the loads, the aeroelastic behaviour of the rotor blades including characteristic coupling effects has to be considered. To determine the properties of the hingeless rotor system in an analytical approach, a mathematical model can be used, which simulates the aerodynamic and dynamic behaviour adequately. There is good experience with an aerodynamically and dynamically equivalent system of an articulated rotor with high hinge offset.

Analytical data as well as flight test data will be shown for different flight conditions including maneuvers. There is relatively good correlation. The loads necessary for the structural design of the rotor can be predicted reasonably well. The methods are not satisfactory for control loads in stalled conditions and for high harmonic vibratory loads.

#### NOTATION

a, a <sub>s</sub> , a <sub>eff</sub>	flapping hinge offset
	flapping coefficient
a <sub>o</sub> , a <sub>1</sub>	number of blades
_	
cg	center of gravity position
c <sub>8</sub>	equivalent flapping hinge restraint
DB	longitudinal stick position
F <sub>Nk</sub>	amplification factor
G, GW	helicopter gross weight
k	harmonic order
М	moment
N	load factor
P <sub>k</sub>	generalized force
R	rotor radius
r/R	blade station
S	shear force
v	forward speed
×s	cg position
ß	flapping angle
ß <sub>k</sub>	feathering axis precone angle
ζ	lead-lag angle
θ <sub>0</sub> , θ <sub>c</sub> , θ <sub>s</sub>	collectiv, cyclic control angle
Ω	rotor angular frequency
ω <sub>β</sub>	flapping natural frequency
ωζ	inplane natural frequency
Ψ	rotor azimuth position

#### 1. INTRODUCTION

At all times the rigid rotor with flapwise and lagwise rigid attachment of the rotor blades was very attractive to the designer because of its highly simplified design compared to the fully articulated rotor. However, in the early days of helicopter development the trials with rigid rotors were practically never successful. The reasons were the insufficient knowledge of the physical-technical correlations and the lack of materials to withstand the high structural loads. In the more recent era a relatively good understanding of the important problems has been gained, and, in addition, suitable materials became available which offer good strength properties also for fatigue loads.

The rigid rotor system allows the transfer of large moments from the rotor to the fuselage which results in a substantial improvement of the flight mechanical behaviour of the whole helicopter. On the other hand, this transfer of large moments through the hub results in different structural requirements compared to articulated rotors.

The main components of the BO 105 rotor system which will be considered (Figure 1) are a very stiff hub and fiberglass rotor blades of high elasticity. There are no flapping and no lagging hinges, the blades are rigidly attached to short hub arms. The feathering axes of the blades are fixed to these substantially rigid hub arms. The blade motions are pitching at the root and bending and torsional deflections. The main features of such a rotor system can be described best by its flapping stiffness resulting in a flapping frequency ratio of about 1.10 to 1.15 and an inplane stiffness with a frequency ratio of about 0.6 to 0.75 for the first inplane mode.

An analytical treatment,  $v^{\pm}$  for loads prediction, of such a rotor needs consideration of its aeroelastic behaviour including the coupling effects due to the elastic blade deflections (1÷3).

#### 2. LOADING CONDITIONS OF HINGELESS ROTOR HELICOPTERS

The main purpose for the introduction of flapping hinges has been the prevention of high moment loadings at the blade roots and the rotor hub. For hingeless rotors without flapping hinges there exists the possibility to transfer high moments from the blades to the hub and the fuselage. Besides of the mechanical simplification this results in improved control and stability characteristics.

The high moment loads at the blade attachment area and the hub have to be considered in the design. Figure 2 illustrates the different loading situation of a hingeless rotor and an articulated rotor with a small flapping hinge offset. If the aerodynamic lift at the blade is the same for both rotors, it produces the same load at the hinge or blade attachment, respectively, and at the center of the hub; for the moments there is a very strong difference at the blade root and the hub. The resulting moment at the center of the rotor can be reduced to a certain degree by preconing the hub arms and thus producing an unloading centrifugal force. Normally, the precone angle will be chosen for unloading with the design rotor thrust. Other thrust conditions and of course alternating thrust of the blades will result in corresponding moments at the hub. The figure shows only the spanwise distributions of the alternating flapping and inplane moments for a typical forward flight condition. The most stressed section of a rigid rotor is the hub and the blade attachment section, which has been anticipated. The blade main section is relatively low stressed, also in comparison to the articulated blade. The flapping moment peak occuring at about 0.7 radius, which is customary with articulated blades, is of much lower magnitude for the rigidly attached blade. The difference is due to the higher harmonic moments. As is shown in the frequency diagram of Figure 3 the rigid blade attachment raises the frequencies of the blade modes. While for the articulated blade there are flapwise modes with frequencies some higher than  $2\Omega$  and  $4\Omega$ , they come closer to  $3\Omega$  and  $5\Omega$  for the rigidly attached blade. The aerodynamic loads are decreasing with increase of harmonic order, therefore resulting by the dynamic amplification in lower blade moments for the rigidly attached blade with natural frequencies near  $3\Omega$  and  $5\Omega$  compared to the corresponding situation with the articulated blade having frequencies nearer to  $2\Omega$  and  $4\Omega$ . A similar situation exists for the inplane moments.

The possibility to transfer high moments from the rotor blades to the hub and the fuselage gives a different situation for the control and trim behaviour compared to helicopters with articulated rotors, as is described in Figure 4. The control of helicopters with articulated rotors is mainly done by inclination of the thrust vector thus producing a moment around the center of gravity. For a helicopter with a hingeless rotor system an inclination of the thrust vector is combined with a very strong hub moment, and the moment around the center of gravity is a combination of the hub moment and the moment due to the thrust inclination. The loading of the rotor shaft and the gearbox with its suspension is different in the two cases. For the articulated rotor the moment is built up linearly from the hub to the center of gravity; in the case of the hingeless rotor the mast and already the hub are subjected to a relatively high moment loading. The capability to produce large moments is much higher for the hingeless rotor, which is the main source for its improved control characteristics.

Trim conditions, which need a rotor produced moment to overcome cg-travel or slope landing conditions, for instance, require an alternating first harmonic moment in the rotating system for the hingeless rotor system, whereas in the case of an articulated rotor because of the equivalence of cyclic control and blade flapping only an inclination of

the thrust is necessary, and the rotor shaft at the hub is subjected to an alternating longitudinal force in the rotating system. For the trim requirements in forward flight there are nearly no differences in both rotor systems, because the cyclic control is needed to overcome an aerodynamic nonuniformity, to which the dynamic systems of the rotors are of lesser importance, see Figure 5.

Higher harmonic blade loads resulting from the flow conditions of the forward flight produce alternating forces at the hub for both rotor systems, and in addition for the hingeless rotor moments at the blade root and the hub. For a dynamically well tuned hingeless rotor these higher harmonic moments are relatively low compared to the first harmonic moments needed for trim or flight maneuvers. Figure 6 shows a typical case with a pronounced first harmonic part. The higher harmonic loads are exciting vibrations; and a low vibration level requires a good dynamic design for the rotor itself and in combination with the fuselage, for all rotor systems.

#### 3. SHORT DESCRIPTION OF CURRENT METHODS FOR THE PREDICTION OF RIGID ROTOR LOADS

The best way to understand the hingeless rotor is to regard it as a successful attempt to produce a rotor whose flapping and lagging hinges are replaced by the elastic deformation of the fiberglass rotor blades. To determine the properties of such a rotor in an analytical approach, a system is to be used which simulates the aerodynamic and dynamic behaviour adequately. For simple studies, it can be assumed that the blades have flexibility only in the flapwise direction and are infinitely stiff in the chordwise and torsional directions. The main features of such a flapping equivalent system are shown in Figure 7. Practically, it is a conventional hinged blade with hinge restraint, simulating the first natural mode and frequency of the real blade, only with a relatively large hinge offset. In Figure 8 the moment distribution over the blade span is illustrated for a typical flight condition. It can be seen that the moment raises mainly in the very stiff parts of the root section and the rotor hub; and it seems well justified to represent this blade configuration by an equivalent flapping system. Experience has shown that this simple equivalent system can model the rotor well enough for stability and control calculations as well as for the prediction of the determining loads including steady-state flight, maneuver and gust conditions.

A more refined equivalent system, as shown in Figure 9, is an extension of the pure flapping system and has additional degrees of freedom in chordwise and torsional direction. This model includes control system flexibility, blade flapwise bending (first mode), blade inplane bending (first mode) and blade twisting (first twisting mode). While inplane bending of the blade has nearly no direct influence on aerodynamic loading and flight behaviour, it may effect the control system dynamics, if control flexibility is considered. Torsionally elastic motions of the blades are included as they act as control inputs and thus can have an influence on aerodynamic loading and on flight behaviour. Effects of different positions of the aerodynamic center, the elastic axis and the center of gravity of the blade profile sections as a function of blade span can be calculated (2; 3).

The aerodynamic model of the rotor is based on current blade element theory using two dimensional airfoil data, considering stall, reverse flow, compressibility effects, yawed flow and unsteady effects. For the aerodynamic model of a hingeless rotor the same assumption as for an articulated rotor can be used fundamentally. For all practical purposes for which only low order rotor harmonic loading is of importance it seems to be sufficient to use simple assumptions for the rotor inflow. The mean induced velocity can be calculated by momentum theory with a trapezoidal distribution in forward flight. The aerodynamic forces and the dynamic response of the rotor blades are obtained by an azimuthal step-by-step computation, solving the equations of motion numerically.

In the MBB-Company this aeroelastic rotor theory is the basis for all flight-dynamics studies as well as for loads prediction. It is completed by an analytical representation of the entire aircraft, including fuselage, tailrotor, tail planes and additional wings or auxiliary thrust if desired. In all calculations the first step is a trimming run, producing the proper control inputs of the rotors and the other trim conditions. The program can be used for steady flight conditions as well as to calculate the dynamic response and loading in maneuvers or gust conditions by a step-by-step computation.

This analytical method shows good correlation between theory and flight test results. Figure 10 illustrates a comparison between measured and calculated hubmoment and root bending moments for a forward flight condition. The moments are shown as functions of rotor azimuth angle. There is good correlation for the low harmonic components. With the assumption of the analytical model no better prediction of higher harmonics can be expected, but on the other side the test results are showing that for the hingeless rotor the higher harmonics are of very low importance. Figure 11 presents maneuver loads in forward flight as a function of load factor. The longitudinal stick position, the mean and alternating flapwise and inplane bending moments are illustrated. The relatively good correlation between theory and test data allows the conclusion that analytical models, as described before, will be well suited for calculation work on hingeless-rotor helicopters, for which mainly first harmonic loading is decisive for component sizing. The main advantage of the method, besides of its simplicity and clearness, is its applicability for unsteady conditions.

The aerodynamic approach for the rotor inflow used in the prescibed aeroelastic

rotor program is a modified actuator disc theory. There is no doubt that there are better aerodynamic theories available, today, but the question arises if more accurate theories can give better results, especially in loads prediction, in the view of engineering approximation. Sometimes, it seems that the importance of improved vortex and wake theories has been overestimated for helicopter design work. No doubt, it is a real scientific task to develop more advanced aerodynamic rotor theories to improve the understanding of the physics of the rotor. Unfortunately, the advanced theories are too complex, and they are difficult in handling, but the most important restriction is that most of them are only working for steady-state conditions, whereas the critical loads are determined by unsteady conditions such as maneuvers and gust penetration, normally.

With an improvement of the aerodynamics, resulting in accurate higher harmonic aerodynamic loads, improved dynamics of the rotor considering higher blade modes should be combined, because only a balanced aerodynamic and dynamic theory can give better results. With the knowledge of today it will be less problematic to refine the dynamic models though there are some problems considering material properties especially damping and nonlinear effects. More accurate dynamic theories will result in computer programs with higher complexity principally, which can be used for steady-state conditions as well as maneuvers, whereas for the aerodynamic refinement real basic problems exist in the case of nonstationary flight.

There are some good reviews about current possibilities to use more accurate wake representation for rotor blade airloads and aeroelasticity (5+7), and it does not seem to be neccessary to repeat them here. For design work, some more attention should be paid to improved theories, which consider only the low harmonic distributions (8).

#### 4. PREDICTION OF PITCH LINK LOADS

There is a special situation with the pitch link loads. Their rapid increase with beginning blade stall at high forward speed or maneuver conditions may be the limitation of the helicopter flight envelope. With a proper component sizing it seems to be not a strength and fatigue problem of the pitch links themselves, but a strong increase of the pitch link loads may be a sign of stall flutter and in any case it signals that the rotor has reached its aerodynamic limitations. Therefore good prediction methods should be available.

Figure 12 illustrates typical pitch link loads by a comparison of test data received with symmetrical and with cambered airfoil sections. Besides of the reduction of loads over the whole speed range for the cambered airfoil caused by a shift of the aerodynamic moment, a still more important effect can be recognized. The increase of alternating pitch link loads with speed and in maneuver flight is reducted due to the more favourable stall behaviour of the cambered airfoil. Figure 13 compares calculated and measured pitch link loads at high forward speeds but without pronounced stall. There is a reasonable good correlation even for maximum speed, with a Mach-number at the advancing blade up to 0.93. The calculations are done with the same aeroelastic model as described in chapter 3 including control dynamics and blade coupling effects. Figure 14 shows measured and calculated waveforms for such a flight condition, which are in very good agreement. The high loads at the advancing blade are due to Mach-number effects.

For a typical highly stalled condition the waveforms are changing as can be seen in Figure 15 comparing unstalled and highly stalled conditions. Without stall the predominant harmonic contents are mainly first harmonic. In the stalled high speed case there are very strong higher harmonic contents resulting from a disturbed blade torsion. Because of the relatively strong coupling effects of a rigidly attached blade of high elasticity accurate prediction of blade moment loads will need an improved dynamic blade model considering all blade modes with frequencies up to the torsional frequency. While such an approach can be done without basic problems, the real uncertainty exists in the aerodynamic model for dynamically stalled airfoils. Although, in recent years, very important research work giving a significant advancement was done, the situation is still insufficient.

#### 5. UTILIZATION OF PREDICTED LOADS IN THE ROTOR STRUCTURAL DESIGN

In all steps of the development of a helicopter, from preliminary design to final certification, it is necessary to have information about the flight loads. In the first steps, as long as the design is not yet defined, more rough information seems to be sufficient; but as soon as the rotor is defined performancewise, a dynamic and structural optimization of the rotor and the rotor blades has to follow. While the fundamental blade flapping mode is determined by the required flight behaviour, there may be design criteria for the first inplane and first torsional blade modes by aeroelastic instability effects. The higher modes should be considered mainly for their influence on higher harmonic loads and vibrations. Resonance diagrams, as shown in Figure 3, are most helpful in the optimization process, in which mainly the dynamic characteristics are envolved. By a dynamic tuning of the blade, using additional masses or changed stiffness, a design should be found for which resonance conditions of the blade frequencies with the rotor exciting harmonics can be avoided. Figure 16 illustrates amplification factors for the blade bending modes by generalized exciting forces of different harmonic order at normal rotor speed. The amplification factors are higher for higher modes, but it has to be noted, that the magnitude of the exciting forces is decreasing with harmonic order. Mainly first

harmonic loads are exciting the fundamental bending mode, second, third and fourth harmonics the first bending mode and fourth, fifth and sixth the second bending mode. By considering more general information about higher harmonic generalized forces, which has been obtained from tests with different rotors showing relatively good conformity, it is possible to look for a good compromise of the blade dynamic design. The variation of the generalized forces with harmonic order is indicated in Figure 17. The decay with harmony order is very similar to that obtained with other rotors. The resulting bending moments are determined by the amplification factors and the shown generalized forces. The outlined approach for the dynamic optimization of rotor blades has proved good since long years (9), and is practicable still today. It does not need much detailed aerodynamic information.

For the initial component sizing for fatigue, the highest level flight loads should be below the endurance limit for infinite life so that sufficient component life will be available to absorb the larger maneuver loads. For other reasons a certain moment of inertia of the rotor is desired; therefore no weight-saving construction is necessary normally. This will result in favourable conditions for the stresses. For the hingeless rotor the determining loads are first harmonic, which can be calculated easily by the methods described in chapter 3. The structural design which is done in an iteration process has to assure that both strength and dynamic constraints are met.

When the design is reviewed and fixed, loads for the complete missions and fatigue spectrum will be calculated. In combination with component fatigue tests, these calculated loads will allow the determination of component lives. As long as the life estimation is done using Miner's theory of cumulative damage, as is still common practice in helicopter engineering, the loads prediction methods described in chapter 3 seem to be well justified and sufficient, because only loads higher than the endurance limit will be considered. But it should be noted, that only loads methods are suitable, which allow the prediction of rotor loads in unsteady flight conditions, such as maneuvers, gusts, etc. The predicted loads also would be acceptable for load spectrum testing.

For the final qualification and certification of a helicopter measured flight loads will be available. With these loads the fatigue spectrum and the determination of component lives will be revised. Only minor corrections will be necessary, if there was adequate work in the previous stages of development.

Helicopter components have long service lives, normally. They come close to unlimited life. Figure 18 shows as an example the fatigue spectrum of a rotor blade. The loads of maximum level flight are lower than the endurance limit, only extreme maneuver conditions result in higher loads thus limiting the life time. Similar relations will be for all components of good structural design. Loads and mission profiles, which will be used normally, are so conservative, that differences in calculation methods and uncertainties in fatigue damage theories will be of little significance. The conservative assumptions make statistically predictable failures extremely improbable (10).

#### 6. CONCLUDING REMARKS

Experience has shown that analytical methods for the prediction of loads are available giving good correlation between calculated and measured loads for level flight as well as for maneuvers. The methods have proved good in the full speed and maneuver range of modern helicopters with moderate advance ratios. The special loading conditions of the hingeless rotor helicopter cause no additional problems of importance, as long as the aeroelastic behaviour of the rotor blades including characteristic coupling effects will be considered. For the prediction of loads it seems to be justified to take care only for the fundamental elastic modes, which can be done by simulating the hingeless rotor by an equivalent articulated rotor with high hinge-offset or by application of the modes directly. Because of the relatively strong control coupling effects, torsional modes should be used in addition to flapping and inplane bending modes. Improvements to the aeroelastic rotor theory by adding higher blade modes would cause no major difficulties but soon could bring the computer programs to practical limits of computer time and numerical tractability.

A major design objective is to produce a helicopter with a flight envelope limited by power and not by structural limits. To be successful it is necessary to have a capability for the analytical prediction of loads for the total envelope, with reasonable accuracy for the highest loads. Deficiencies still existing for highly stalled conditions can be reduced by the inclusion of available unsteady stall aerodynamics in the computer programs. Additional research work should be accomplished.

The described loads prediction methods can give nearly all information necessary for structural designing for which only the highest loads will be considered, normally. For a dynamically well optimized rotor the critical loads are low harmonic loads, for a hingeless rotor mainly first harmonic loads. Higher harmonic loads are not of importance for the structural design but they decide the vibration characteristics of the whole helicopter. Further improvement of vibration predicting techniques still seems to be necessary utilizing improved aerodynamics as well as an improved dynamic representation of the whole helicopter. Suitable routines can be limited to level flight mainly, thus allowing much more complex mathematical models. A helicopter with a low vibration level in normal flight conditions will have good characteristics during maneuvers also.

#### 7. REFERENCES

- 1. Reichert, G.: "The Influence of Aeroelasticity on Stability and Control of a Helicopter with a Hingeless Rotor", AGAFD-CP 46, 1969
- Reichert, G.; Huber, H.: "Influence of Elastic Coupling Effects on the Handling Qualities of a Hingeless Rotor Helicopter", 39th AGARD Flight Mechanics Panel Meeting, Hampton, Virginia, September 1971
- Huber, H.: "Some Objectives in Applying Hingeless Rotors to Helicopters and V/STOL-Aircrafts", AGARD-CPP 111, September 1972
- Reichert, G.: "Review of German Activities in Helico; ter Structures and Dynamics", Working Group on Helicopters-V/STOL Aircraft Structures & Dynamics, 31st AGARD Structures and Materials Panel Meeting, Tonsberg, Norway, November 1970
- Berrisford, R.S.: "State-of-the-Art Survey of Loads Prediction, Experimental Methods and Failure Criteria", Working Group on Helicopters-V/STOL Aircraft Structures & Dynamics, 32nd AGARD Structures and Materials Panel Meeting, London, England, March/April 1971
- Joklekar, M.; Loewy, R.: "An Actuator-Disc Analysis of Helicopter Wake Geometry and the Corresponding Blade Response", USAAVLABS Technical Report 69-66, AD 881 981, December 1970
- Sadler, G.: "Development and Application of a Method for Predicting Rotor Free Wake Positions and Resulting Rotor Blade Air Loads", NASA CR-1911, December 1971
- Ormiston, R.A.: "An Actuator Disc Theory for Rotor Wake Induced Velocities", AGARD-CPP 111, September 1972
- 9. Daughaday, H.; Kline, J.: "An Approach to the Determination of Higher Harmonic Blade Stresses", American Helicopter Society Proceedings 9th Annual Forum, 1953
- 10. Peck, W.B.: "A Survey of Helicopter Current Practices Relative to Fatigue", Working Group on Helicopters V/STOL Aircraft Structures & Dynamics, 32nd AGARD Structures and Materials Panel Meeting, London, England, March/April 1971

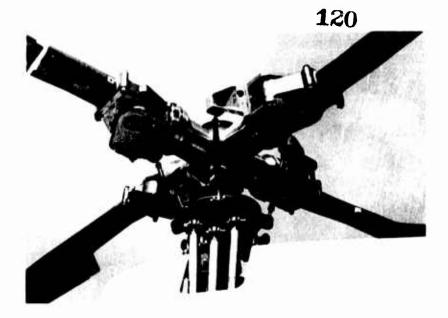


Fig. 1 Rotorhub and Blade Attachment of the BO 105 Hingeless Rotor System

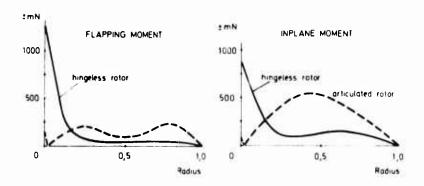


Fig. 2 Comparison of Alternating Bending Moments for the Hingeless and the Articulated Rotor

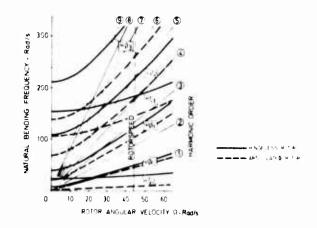


Fig. 3 Rotor Resonance Diagram

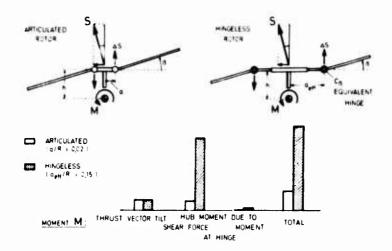


Fig. 4 Rotor Control Moment Capability

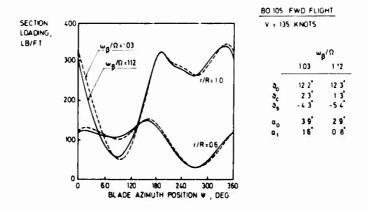


Fig. 5 Blade Section Loading and Trim Values with Different Rotors

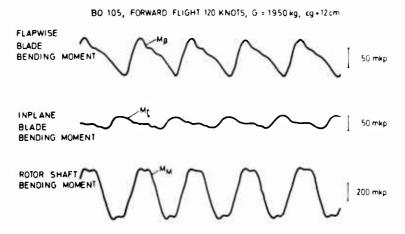


Fig. 6 Typical Flight Loads of a Hingeless Rotor

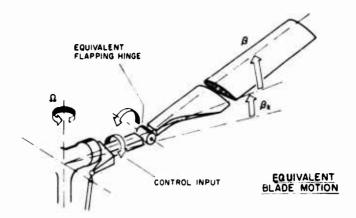


Fig. 7 Analytical Model of Rotor Blade

## FLAPWISE BENDING MOMENT

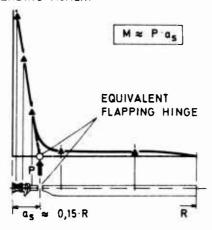


Fig. 8 Distribution of Flapwise Bending Moment on a Rigid Rotor Blade

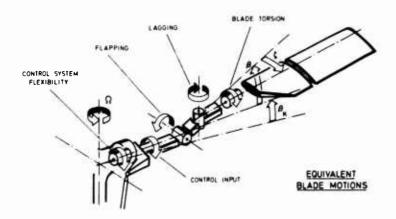


Fig. 9 Analytical Model of Rotor Blade with Elastic Flapping, Lagging and Torsional Degrees of Freedom

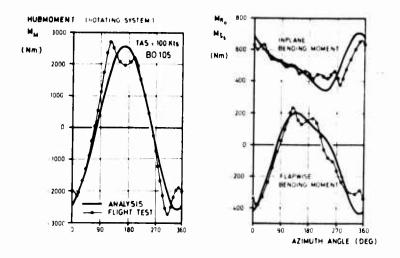


Fig. 10 Hub- and Blade Root Moments in Forward Flight

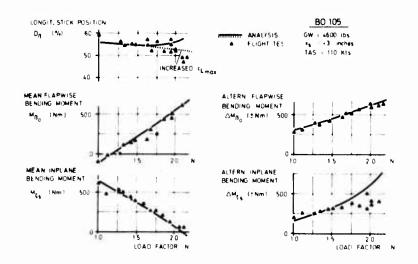


Fig. 11 Maneuver Loads in Forward Flight

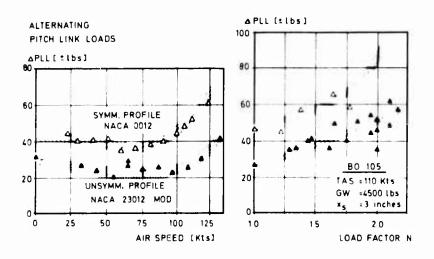


Fig. 12 Alternating Pitch Link Loads in Forward Flight and Maneuvers

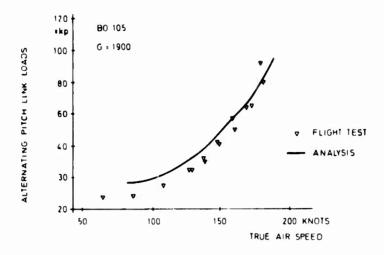


Fig. 13 Pitch Link Loads vs. Air Speed for Extreme High Speeds

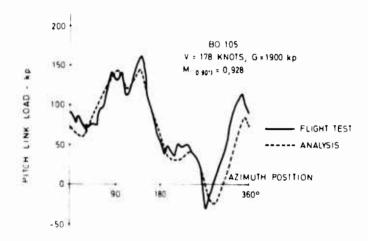


Fig. 14 Pitch Link Load Waveform

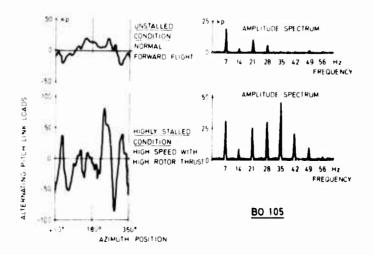


Fig. 15 Pitch Link Loads Unstalled and Stalled Conditions

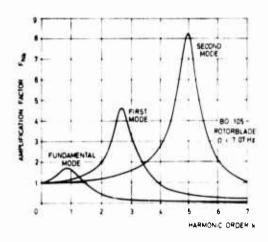


Fig. 16 Amplification Factor vs. Harmonic Order

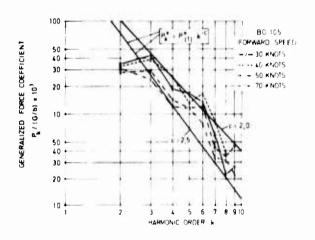


Fig. 17 Generalized Force Coefficient vs. Harmonic Order

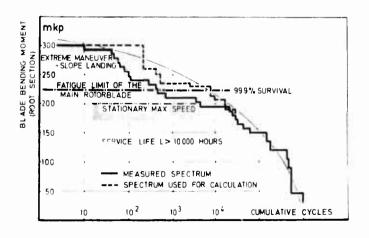


Fig. 18 Flight Spectrum BO 105-Rotor Blade

#### INTEGRATED ROTOR/BODY LOADS PREDICTION

bν

R. M. Carlson
U.S. Army Air Mobility R&D Laboratory
Headquarters
Moffett Field, California USA 94035

å

A. W. Kerr Lockheed California Co. Burbank, California USA 91503

#### SUMMARY

An interdisciplinary analysis, which has grown out of a requirement for a nonlinear handling qualities evaluation tool, has been mechanized in a fashion which provides a capability to predict rotor loads affected by rotor/airframe interaction in steady-state and transient flight conditions. The modeling philosophy in developing this analysis combines the capabilities of a team of analysts from several specialties to create a versatile model which provides consistent data for numerous applications. This philosophy is presented in addition to a description of the model and a summary of its range of applications. Three examples involving rotor loads prediction are presented: 1) evaluation of clearance between rotor blades and fuselage during extreme maneuvers, 2) estimation of four-bladed rotor reactionless implane mode stability and loads, and 3) general maneuver capability and transient loads estimation. Also presented are areas proposed for continued development and refinement of the model to further increase its range of applications.

#### SYMBOLS

c	rotor blade chord	ß	aircraft sideslip angle
h	aircraft altitude	$\beta_1$	rotor blade 1st flap mode displacement
p	aircraft roll rate	β2	rotor blade 2nd flap mode displacement
q	aircraft pitch rate	βsha	ft shaft bending angle
r	aircraft yaw rate	$\epsilon_1$	rotor blade inplane mode displacement
u	aircraft longitudinal velocity	θ	aircraft pitch attitude
v	aircraft lateral velocity	θВ	blade element pitch angle
w	aircraft vertical velocity	$\theta_{\mathbf{g}}$	control gyro pitch attitude
w <sub>i</sub>	induced velocity at blade element	ρ	air density
x	% rotor radius	φ	aircraft roll attitude
		ф <sub>g</sub>	control gyro roll attitude
		ψ	aircraft yaw angle

## INTRODUCTION

Analysis techniques for estimating rotor and total vehicle <u>stability and response</u> characteristics have traditionally been developed by starting with a minimum number of degrees of freedom to describe the system and expanding simple, low-order analyses to include sufficient additional degrees of freedom to match observed flight phenomena. Rotor blade load analyses have, on the other hand, included a very complete description of blade mass, balance, and stiffness properties which provide detailed load distributions but, due to practical size and computation time constraints, preclude the inclusion of a description of a body and its interaction with the rotor. Although such analyses may include the capability to approximate body response effects on rotor loading, the capability to simulate the total aircraft response and rotor loads due to a given pilot action is not practical. In addition to a detailed physical description of the rotor, traditional rotor loads analyses generally include the most advanced distributed inflow models. These models contain detailed wake representations which increase the computation time by a significant amount.

In undertaking the task to develop a nonlinear <a href="handling qualities">handling qualities</a> analysis for high performance compound helicopters, it became evident that the level of sophistication which appeared to satisfy the analysis needs for that engineering discipline was sufficient to provide good quality data in the areas of <a href="performance">performance</a>, rotor low-frequency structural dynamic stability, and steady-state and transient loads. By definition, this analysis contained an integrated rotor/body representation. The rotor blade motion was described by a set of prescribed primitive modes obtained from a traditional 150 degree of freedom blade loads model, computed with rotor rotation but without airloads. Airloads were generated in the analysis through strip integration based on 1) an emperically modified uniform inflow which is a function of rotor advance ratio and shaft moment, and 2) nonlinear blade section aerodynamic characteristics dependent on angle of attack, Mach number and distributed blade section thickness ratio and camber.

At the time, the basic analysis represented the work of a single individual with a view to needs in his own specialty. Another analyst, whose interest was primarily in loads prediction, modified the analysis independently to include a capability for harmonic analysis of loads as well as several other improvements. Soon other specialists were also creating their own modifications, often without a thorough understanding of the analysis that they were modifying. In all areas, results of the analysis were interesting, but soon it was obvious that all versions were not consistent, and data management was unwieldy. A requirement to restructure the analysis was recognized and a program was undertaken to use the experience gained in the prior development activities to orient the restructuring. The approach used has been presented in depth in reference 1, and is briefly summarized here.

#### ANALYSIS PHILOSOPHY

The approach taken in the restructured development of the analysis was to form an analysis project team composed of a specialist in each engineering discipline and an overall analysis manager. The purpose of the management task was to assure that the analysis contained the most complete and up-to-date analytical description possible in each specialty within practical program size and execution time constraints. The manager does not write the analysis; he assures that all elements of the analysis are completely discussed and understood by the team; that a consensus is reached for proceeding, and that the analysis is programmed and checked completely. As a result, the analysis does not reflect a single specialist's view of the state of the art and provides an analysis team, rather than one individual, with an in-depth understanding of an extensive and complex mathematical model, its area of application and its limitations.

The model describes the entire dircraft with considerable detail in the representation of the rotor and control system. A maximum number of significant degrees of freedom is selected at the outset to avoid a common failing of the traditional analysis of approach, i.e., the discovery of dynamic phenomena in flight rather than in analysis before flight. Once the extent of the model is defined, the dynamic equations of motion are derived completely in a Lagrangian energy balance form with an absolute minimum of simplifying assumptions. Small angle assumptions are avoided completely at this point in the derivation and all equations are derived in matrix form. The matrix expressions of the analysis are programmed in modules and these same expressions are expanded through a computer technique called FORMAC to provide algebraic equivalents which are programmed and checked against the matrix modules. Simplifying assumptions and decisions on the elimination of terms are made systematically after the total analysis is completed. Other elements (i.e., tail rotor, propeller, etc.) of the analysis are programmed and checked in modular form and the interfaces between modules are programmed and checked.

An element of major importance in implementing an extensive analysis of this type is data management. A method must be established to assure that whenever the analysis is executed, there is a means of checking input data and assuring that output options are sufficiently clear to avoid costly aborted computer runs.

The matrix form of the equations of motion is designed to provide an additional analytical capability. The equations of motion may be perturbed to provide constant and periodic linear coefficients which can be used in linear analyses and to check linear analyses derived by other methods. In this way the large, complete model generates a consistent set of simpler analyses which can be used to study various flight conditions in depth at a substantially reduced analysis cost.

#### MODEL DESCRIPTION

The model which resulted from the foregoing philosophy has been written for a single four-bladed, gyro-controlled hingeless rotor helicopter with additional capability for analysis of teetering or hinge-offset rotor systems with conventional controls. This aircraft may be conventional in design, winged or compounded. The specific analysis is limited to a maximum of four blades; however, the analysis can be expanded to include more blades by following the detailed mechanical derivation procedure established for the analysis. The model is divided into three major categories shown in Fig. 1. These categories are related to one another, as utilized in the analysis. The analysis is the simulation of an entire rotor-craft which includes a detailed dynamic description of the rotor and control system as well as the conventional six degree of freedom body dynamic description which operates in two modes identified as TRIM and FLY. In the TRIM mode, the aircraft is constrained to a prescribed static flight condition while the controls are activated to obtain a force and moment equilibrium of the aircraft at that static condition. In the FLY mode the rotor, controls, and airframe are free to respond dynamically to control or external inputs.

A more detailed logic flow diagram of the analysis is presented in Fig. 2. Identified in this diagram are all of the major elements of the analysis. The initial information required includes a detailed physical description of the entire rotorcraft, specification of initial flight conditions at which the analysis is to be performed, and any pilot or external input desired during the FLY mode. Once this information is provided, all of the equations are set up with the proper initial values and the analysis proceeds in the TRIM mode. In the TRIM mode, the body and the rotor speed accelerations are not included as part of the solution of the equations of equilibrium, and the loading and motions of a single blade are computed to reduce computation time. The trim procedure which adjusts basic control and body attitudes required in a steady flight condition, is utilized to iterate to a trim condition. This procedure operates directly on main rotor collective and cyclic pitch, tail rotor and propeller collective pitch, and aircraft attitude. Trim may be established either for a straight and level flight condition or for a constant load factor maneuver. When TRIM has been completed, the analysis proceeds to the FLY mode where all degrees of freedom which include four independent blades, are activated and the aircraft responds for a specified length of time to any desired pilot or external input. Pilot inputs can be simple steps or pulses in any single control axis, doublets, stick stirs or any other transient input within the capabilities of the control system simulated. As a result, transient loads and resulting aircraft and rotor dynamic response can be obtained. For correlation purposes, actual flight test control motions can be input to provide comparative response data. For specialized applications, an analytic autopilot may be used to control the flight path of the aircraft.

The rotorcraft is described dynamically in thirty fully-coupled degrees of freedom. In addition to the normal six body degrees of freedom, the rotor hub is described with pitch, roll, and height displacement (shaft deformation) as well as rotational speed for a total of four degrees of freedom. The control gyro/swashplate combination also has the same four degrees of freedom. Motion of each of the four main rotor blades are described by two flapwise and one inplane modes and a pitch horn bending degree of freedom which couples blade feathering to the control gyro. These four degrees of freedom per blade make sixteen for the four blades, bringing the total system to thirty degrees of freedom. A schematic description of the degrees of freedom are shown in Fig. 3. In addition to these dynamic degrees of freedom, there is an approximation of a 1st torsion mode for each blade. Since the frequency of this mode is usually high (over  $4\Omega$ ) compared to the other dynamic modes of interest, a full dynamic representation of this mode would increase the computation time by an order or magnitude. This mode is included as a massless elastic response to blade torsion moments with a first order lag. It has been found that this allows the blade to respond to provide more realistic airloads distributions.

The dynamic equations of motion in thirty degrees of freedom are written in matrix form as:

$$- [A] \{a\} + \{G\} = 0$$

where [A] is a 30  $\times$  30 matrix of generalized mass elements,  $\{\ddot{q}\}$  is a column matrix of accelerations of the generalized coordinates, and  $\{G\}$  is a column matrix derived from the Lagrangian Energy Functions, dissipation function and generalized forces, which takes the form:

$${G} = -[B]{d} - [C]{q} + [Q]{f(t)}$$

The equations of motion are solved as a time history at rotor azimuth angles required to provide a stable solution for the highest frequency mode present in the solution.

The blade modes are primitive modes in that they are determined from a lumped parameter analysis of a rotating cantilever blade at a selected rotor speed and collective blade angle. The generalized stiffness matrix is computed using these rotating modes and contain only the structural stiffness of the blades and hub. This formulation ensures proper internal and external force and moment balance. The modal deflections outboard of the feather hinge are rotated through the actual feather angle less the reference feather angle. Thus, blade element deflections outboard of the feathering hinge due to modal displacements are defined to remain aligned with a coordinate axis system which is orthogonal to a plane containing the instantaneous deformed feather axis and rotated through the instantaneous feather angle less the reference feather angle. Thus, the internal strain energy in the blade due to unit modal displacemental is invariant with collective blade angle. This technique permits the highest resolution of motion and forces for the blade with an assumed mode solution for a given number of modes.

Since the blade element deflections outboard of the feathering hinge are rotated through the feather angle, a new generalized mass statement is required at each instant in time but it permits an exact description of the relationship between the line of action of the centrifugal forces and the orientation of the blade element structural principal axes. In addition, since the modes being used are based on the rotating beam, fewer modes are required to achieve accuracy in the predicted dynamic response of the blades. This technique results in a coupled stiffness matrix between the modal deflections whereas conventional methods using a rotating invariant normal mode representation would result in a diagonal stiffness matrix for the blade.

In a rotor simulation of this type, it is difficult to compute the proper displacement velocities and accelerations, and associated inertia and aerodynamic forces and moments which are required for high resolution of the blade feathering moments. This requires exacting aerodynamic data as well as a precise statement of the inertial loadings. To establish the feathering moments due to these loads, the relationship between the feather axis and the point of application of the loads must be precisely determined. This is accomplished by a very accurate analytic construction of the undeformed blade and a superposition of the blade elastic bending on this shape. In order to achieve the highest resolution to the predicted blade shape and feather axis position, the blade modes are defined at approximately the trim collective blade angle. The blade static position is also constructed at this blade angle.

The aerodynamic description employed in the analysis is composed of a rotor inflow model, nonlinear steady and unsteady blade element aerodynamics, nonlinear body aerodynamic characteristics, rotor/body aerodynamic interference, and auxiliary airloads from the tail rotor and propeller. The auxiliary airloads are contained in modular subroutines and are functions of advance ratio and propeller and tail rotor collective pitch. The main rotor downwash effect on the wing and horizontal tail angles of attack is an empirical function of rotor thrust and advance ratio based on data from reference 2 and similar sources. The nonlinear body aerodynamics may be input as tables containing actual wind tunnel test data.

The rotor inflow model used is an empirical modification to uniform momentum downwash based on data from reference 3 with adjustments for shaft moments. The inflow at a blade element is of the form:

$$w_{i} = \overline{w}_{i} + 1 + x \left[ f(X_{i}) \cos \psi + f(X_{v}) \sin \psi \right] + x \left( \overline{p}_{i} \sin \psi + \overline{q}_{i} \cos \psi \right)$$

where  $\overline{w_i}$  is the uniform momentum inflow,  $f(X_{u_i,v})$  are functions of longitudinal and lateral wake angles, and  $\overline{p_i}$  and  $\overline{q_i}$  are functions of rotor roll and pitchin, moments and translational velocity.  $\overline{w_i}$ ,  $\overline{p_i}$  and  $\overline{q_i}$  are filtered with first order lags that represent the delay in establishing the inflow following a change in rotor loading condition.

Blade section aerodynamic lift, drag, and pitching moment are nonlinear functions of section thickness ratio, camber, angle of attack, and Mach number. Blade element unsteady aerodynamic effects due to pitch and plunge velocity are quasi-steady with a Theodorsen deficiency function of 1.0 and are of the general form round in reference 4. These corrections for unsteady aerodynamic effects are added to the steady airloads and are implemented in the following form at each blade element:

Normal Force = 
$$-\frac{\pi}{4} \rho C^2 \left[ \dot{\mathbf{U}}_{\mathbf{n}} + \frac{\mathbf{C}}{4} \dot{\dot{\mathbf{u}}}_{\mathbf{B}} \right]$$
  
Feather Moment =  $-\frac{\pi}{4} \rho C^3 \left[ \frac{\dot{\mathbf{U}}_{\mathbf{n}}}{4} + \frac{\mathbf{U}_{\mathbf{C}}}{4} \dot{\dot{\mathbf{e}}}_{\mathbf{B}} + \frac{3}{32} \dot{\mathbf{C}} \ddot{\dot{\mathbf{u}}}_{\mathbf{B}} \right]$ 

where  $\mathbf{U_n}$  is the local instantaneous normal velocity of the blade element,  $\mathbf{U_c}$  is the local instantaneous chordwise velocity at the blade element, and  $\mathbf{\theta_B}$  is the blade element pitch angle.

The rotorcraft primary control systems are simulated from the pitch control levers through the boost system in all control axes. Gearing and gains in the control path are inputs to the analysis and may be easily changed for studying the effects of design changes in the control system. Control servos are simulated by first order lags with rate limits and soft and hard physical stops. Control stiffnesses in collective and cyclic pitch axes of the main rotor are included in the dynamic equations of motion.

The primary output is in the form of a time-history solution of the equations of motion. The standard output format provides plots of up to 40 output parameters in TRIM and 60 in FLY. In addition, a complete set of digital output parameters is provided at the end of each mode of operation. Typical FLY output plots are shown later in the example applications. The analysis also provides plot capability at the end of TRIM to show loads at various points in the system over a single rotor revolution on an expanded linear scale. These loads are harmonically analyzed and the harmonic components printed out.

As the FLY mode is initiated, an option is available to perturb all thirty dynamic degrees of freedom at all azimuth positions over a full rotor revolution to provide linear coefficients at each point in time. These linear coefficients are then harmonically analyzed and the components form a set linear matrices for linear analysis of phenomena near the selected flight condition. Standard linear analyses can be accomplished by using the steady components or the harmonics may be used to perform Floquet analyses similar to that of reference 5. Both types of analyses provide Eigenvalues to assess system stability and frequencies.

It is often difficult to establish the stability level of a single mode from a complex time-history response. In order to obtain a measure of damping of a specific mode from a channel of data, a moving block fast fourier transform (FFT) technique is used to analyze the same data output for automatic plotting. Response data like that shown at the top of Fig. 4 is analyzed by taking a small block, or time interval, of the data and transforming it to obtain the response amplitude at each frequency in the data. This transform is plotted as shown at the bottom of Fig. 4 for one block location. The block is then moved repeatedly to start at later times and transformations are made producing a family of transform plots. The computer then takes specific peak frequencies from the series of transforms and plots the log of the amplitude as a function of time, producing output like that shown in Fig. 5. The damping of the frequency of interest is obtained by taking the slope of time plot for that frequency. For the case shown, the positive slope agrees with the unstable nature of the time history.

#### LOAD PREDICTION APPLICATIONS

The analysis as described can obviously provide rotor loads information at the end of the TRIM mode for steady-state flight conditions in the same manner as conventional steady-state rotor loads analysis. The unique aspects of the method lie in the ability to perform analysis of loads related phenomena which cannot be accomplished without the full aircraft description. Three applications which illustrate many of the features of the analysis have been selected as examples. The first of these applications was an investigation to determine the degree of clearance between the main rotor blades of the AH-56A compound helicopter and the cockpit canopy during extreme control applications. As can be seen from Fig. 6, the normal clearance between the rotor and airframe of this aircraft appears to be relatively low; however, the dynamic characteristics of the hingeless rotor are such that the clearance provided is adequate for all normal flight conditions and the most violent pilot initiated maneuvers. Preliminary analysis showed that the clearance between the blades and the canopy is more critical than that between the blade tips and the tail cone of the fuselage. Blade bending data from the analysis and whirl tower tests were compared and showed close agreement, lending credibility to the analytical results. Figure 7 presents a typical set of time-history results indicating blade/canopy clearance for very violent pilot inputs. In this example, the pilot pushes the cyclic control stick forward to its stop at the cyclic servo rate limit while simultaneously commanding full left roll. This combination of forward and left cyclic pitch inputs were found to be the most critical cases for clearance. On the same figure, the associated load factor, aircraft pitch and roll response, and shaft moments are shown. These are just eight of the 42 channels of information available from the analysis for this maneuver case. Figure 8 presents a summary of several of the different cases examined in the study showing the effects of flight velocity, rotor rpm, trim collective pitch and roll input on rotor/canopy clearance.

A second application of the integrated rotor/body analysis involves a rotor inplane reactionless mode. This dynamic phenomenon first became evident when the reactionless mode was encountered in flight and produced chordwise blade loads sufficient to buckle a blade. The blade motions associated with this mode are shown in Fig. 9, a schematic of a four-bladed rotor. This mode provides no cue to the pilot when it is excited since adjacent blades always move in opposite directions, maintaining a fixed location of total rotor blade system center of gravity. The inplane response is driven at the frequency of the first inplane bending mode (1.30) through (pitch-lag) coupling. Flapping response is also driven at the same frequency as the blades flap up with forward motion and down with aft. Therefore, this flapping motion is also reactionless since it is balanced across the rotor and no net loads are transmitted to the rotor shaft to produce aircraft response.

When the reactionless mode phenomena first occurred in flight, only two of the rotor blades contained chordwise instrumentation and one blade contained flapwise instrumentation. Thus, it was not possible, initially, to identify the fundamental nature of the dynamic phenomena. Study of the flight test collective

control loads indicated significant vibratory loadings at a frequency of  $2.6\Omega$ , or twice the chordwise natural frequency, while the cyclic control was free of this frequency. This observation, when combined with the chordwise and flapwise data, resulted in the conclusion that the phenomenon was indeed a reactionless one. Figure 10 presents analysis results, for a stable case, which reproduce the  $2.6\Omega$  loads in the collective control system. Note also the analysis reproduction of "free play" in the collective system. The effects of this characteristic on stability were being investigated in this particular analysis run.

The stability of the inplane reactionless mode is highly dependent on rotor loading and flight condition. It was encountered in a long duration 20 knot air taxi condition in very calm air at a higher than normal gross weight. The analysis was conducted at the above flight condition and it predicted the presence of the reactionless mode and closely approximated the level of damping observed. Figure 11 presents the damping of the mode as a function of gross weight. At a weight of 18,500 lbs, test and analysis show close agreement and at 20,500 lbs the analysis is slightly conservative. The analysis indicated that damping could be increased over a wide range of gross weights by increasing built-in blade droop between the blade axis and the feathering axis. This relationship was substantiated in subsequent flight testing. Figure 12 indicates the character of the damping of the mode with speed which shows a marked decrease in damping at low speed in both test and analysis results.

Figure 13 presents the inplane loading in the reactionless mode following stick stirs at several flight conditions and the moving block fourier transform analysis of this response showing relative damping.

The third example is a study which was performed for the U.S. Army under Contract DAAJOs-70-C-32 for USAAVLABS reported in reference 6. This study was designed to evaluate maneuverability effects on rotor/ wing design characteristics. For this study, the analysis was used to perform a series of transient maneuvers with several conventional and winged helicopter configurations in order to evaluate load factor capability and associated blade loads. Figure 14 presents a description of the type of maneuvers analysed. These were to be flown in coordinated turns, pullups, and push overs. In order to accomplish these maneuvers, an analytic autopilot is required to represent the pilot flight-path control in executing the maneuvers. The autopilot provides rational control system displacements and corresponding maneuver command inputs and aircraft responses and performs the maneuvers in a manner consistent with good pilot technique. A block diagram of the autopilot used is shown in Fig. 15. The autopilot utilizes body pitch, roll and yaw rate and attitude, height, vertical velocity and normal acceleration signals for adjusting the primary controls to follow the predetermined flight path. The A blocks are gains while the B blocks are first order lags that match pilot stability and response characteristics for accomplishing the maneuvers. The purpose of the analytical autopilot is to simulate pilot control of aircraft flight path and to provide a simple command history format rather than that of a true automatic pilot to perform the usual pilot relief functions.

Using this flight path control technique, the maneuvers were successfully performed and blade loads estimates were obtained at several blade stations for the full range of conventional and winged helicopter maneuvers.

#### CONCLUSIONS

An analysis technique has been developed for use in providing data for several engineering disciplines using a somewhat unconventional modeling philosophy. This analysis has been used in several applications which involve rotor blade-load assessments and show correlation with test data. The analysis has been shown to be a useful tool and several areas for further development should be pursued. In particular, the aerodynamic description can be improved in several areas. The model as currently formulated contains an empirical description of the rotor downwash on the wing/fuselage and horizontal stabilizer. The effect of the flow field around the body on the rotor has not yet been included. A first step would be to include the influence of wing vorticity on the inflow of the rotor. Wind-tunnel measurements of body induced flow in the plane of the rotor disc could also be included. Although a full wake vortex inflow analysis would be impractical, a distributed inflow at a trim condition could be obtained from a wake vortex model and perturbed by the same technique currently used. A further refinement of blade element unsteady aerodynamics would be desirable as well as the inclusion of hysteretic blade-stall effects.

#### REFERENCES

- Kerr, A.W., Anderson, W.D. & Potthast, A.J.: An Interdisciplinary Approach to Integrated Rotor/Body Mathematical Modeling. AHS Symposium on Advanced (Non-Flight) Testing and Modeling Techniques for V/STOL, Philadelphia, Penna., October 1972.
- Bain, L.J. and Landgrebe, A.J.: Investigation of Compound Helicopter, Aerodynamic Interference Effects, Stratford, Conn. USAAVLABS Technical Report 67-44, Contract DA44-177-AMC-203(T), November 1967.
- 3. Heyson, H.H. and Katzoff, S.: Induced Velocities Near a Lifting Rotor with Non-Uniform Disk Loading, NACA Report 1319, Langley Aeronautical Laboratory, Langley Field, Va., 1957.
- 4. B. Bisplinghoff, R. L. Ashley, H., and Halfman, R.L.: Aeroelasticity, Reading, Mass., Second Printing, November 1957, Addison-Wesley Publishing Co., Inc.
- 5. Gockel, M.A.: Practical Solution of Linear Equations with Periodic Coefficients, J. Am. Helicopter Soc., volume 17, Number 1, January 1972.
- Gorenberg, N.B. and Harvick, W.P.: Analysis of Maneuverability Effects on Rotor/Wing Design Characteristics, Lockheed Report 24051, USAAVLABS Contract DAAJ02-70-C-0032, March 1 71.

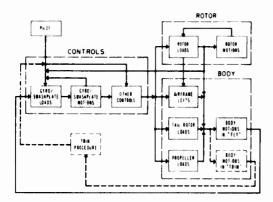


Fig. 1. Basic analysis organization

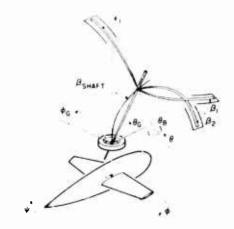


Fig. 3. Analysis schematic

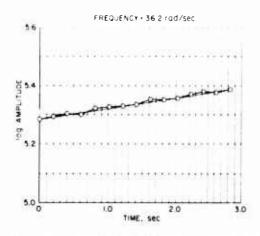


Fig. 5. Typical mode amplitude growth plot

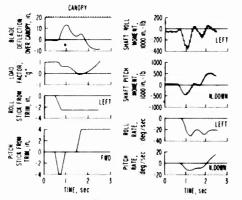


Fig. 7. Canopy clearance time history-V=200 knots

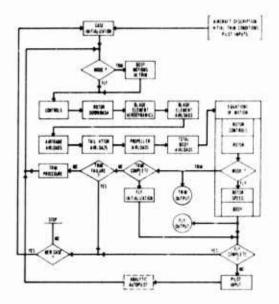


Fig. 2. Analysis logic flow diagram

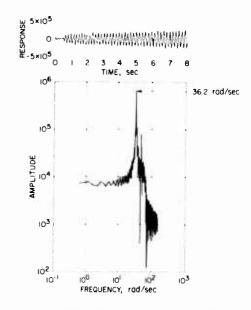


Fig. 4. Typical frequency response output

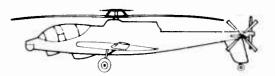


Fig. 6. AH-56A sideview

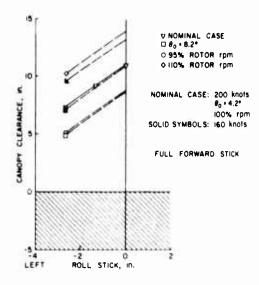


Fig. 8. Canopy clearance analysis summary

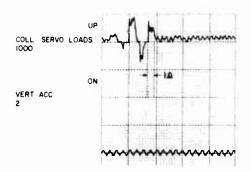


Fig. 10. Reactionless mode collective control loads

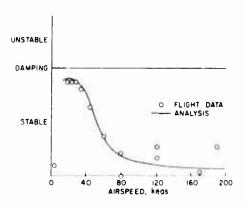


Fig. 12. Effect of speed on damping-W=18,500 lbs

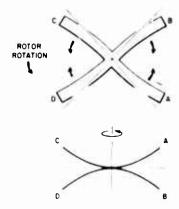


Fig. 9. Reactionless mode blade motions

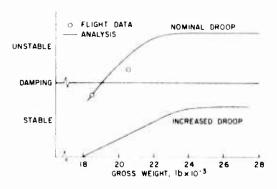


Fig. 11. Effect of gross weight on damping-V=20 knots

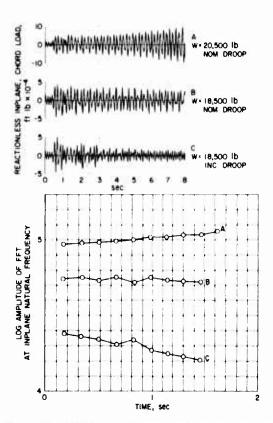


Fig. 13. Effect of droop on damping and chord load

#### MANEUVER SECHENTS

- FILOT RESPONSE LAG TIME . 0.7 sec
- 2 ARCRAFT RESPONSE LAG TIME MITERVAL BETWEEN CONTROL MPUT AND AIRCRAFT RESPONSE
- 3 LOAD FACTOR BUILDUP TIME FINE BETWEEN INITIAL AIRCRAFT RESPONSE AND ATTAINMENT OF LOAD FACTOR
- 4 MANEUVER EXECUTION TIME INTERVAL DURING WHICH LOAD FACTOR EQUALS OR EXCEEDS SPECIFIED LEVEL, APPROXIMATELY 3 sec
- 5 RETURN TO LEVEL FLICHT TIME TO RETURN TO LOAD FACTOR OF 1.0

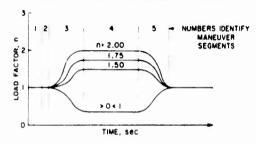


Fig. 14. Maneuver profile

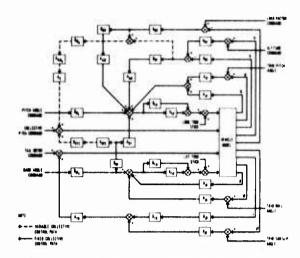


Fig. 15. Analytic autopilot block diagram

#### Review of:

Reichert, G. "Load Prediction Methods for Hingeless Rotor Helicopters"

and

Carlson, R. M. and Kerr, A. "Integrated Rotor/Body Loads Prediction"

bу

#### K. H. Hohenemser

Reichert reports on hingeless rotor load prediction methods developed at Messerschmitt-Bölkow-Blohm (MBB). Carlson and Kerr present a summary of the methods developed for the same purpose at the Lockheed California Company. Off hand it would seem that the mathematical models developed at MBB and at Lockheed are drastically different. The MBB model uses rigid straight blades hinged at the root with a flapping hinge, a lag hinge, a torsion hinge outboard and a torsion hinge inboard of the flapping and lag hinges, all hinges having elastic restraints. In addition, collective and cyclic control flexibility is considered. In a 4-bladed rotor this leads to 12 rotor degrees of freedom and in case of a rigid body with 6 more degrees of freedom to a total of 18 degrees of freedom.

Lockheed's Rexor program uses 150 degrees of freedom for each blade to obtain the normal blade modes including effects of rotation but excluding aerodynamic effects. In the final analysis only 2 flap-bending modes, one lag-bending mode and one torsion mode per blade are retained. Except for the additional degrees of freedom of the Lockheed gyro control system which are absent in the MBB hingeless rotor, the Lockheed Rexor program adds the second blade flap bending mode to the modes used in the MBB program. It also considers the curvature of the blade modes which is neglected in the MBB program. The aerodynamic assumptions in both programs are rather similar: a relatively crude inflow model, quasisteady aerodynamics, large angle assumptions for blade pitch, interblade aerodynamic coupling, look-up tables for compressibility and stall effects. Neither program considers hysteretic stall effects. Using a different degree of freedom for each blade makes for rather long computer runs to obtain a trim condition. Both companies are using, therefore, an abbreviated trim procedure based on the assumption that all blades perform the same motion with an appropriate phase shift. There is a difference in the method of derivation of the basic equations. The MBB program used force balance equations, while Rexor used energy experssions and associated Lagrange equations. The first method has the advantage of easier tracing of the origin of the various terms, the second method has the advantage of a better assurance that all significant terms are included.

The MBB mathematical model, in spite of its rather drastic structural simplifications, can certainly be expected to give a good account of the flight dynamics problems. As far as blade stresses are concerned it seems to provide reasonably accurate maximum stresses in the special case of the B-105 rotor which has constant thickness and constant chord blades, for which maximum stressing occurs at the blade root. For tapered in thickness blades with their greater curvature and their increased stresses at further outboard blade stations, the MBB model is most likely inadequate to predict maximum blade stresses. Reichert made a good point in stating that aerodynamics theories currently available are too complex and not sufficiently substantiated to replace the classical quasisteady aerodynamic assumptions and that consequently the rather crude structural representation of the rotor matches the rather crude aerodynamics. There are, however, definite limitations and qualifications in the use of the straight blade analytical model, some of which will be briefly discussed.

### Second and Third Harmonic Blade Loads

The data presented by Reichert shows that the second blade flap bending mode has a resonance factor of 4.7 at 2.7 P. Ignoring the resonance factor as in the straight blade analysis, the second and third harmonic blade response must be expected to be considerably in error. This error may be small at the blade root, but is substantial further outboard and may be unacceptable for tapered thickness blades. In addition the curvature of the first flap-bending mode will have increasing effects at increasing advance ratio. It would be interesting to compare the results of a blade loads analysis including first and second elastic blade modes with the results of the straight blade analysis.

#### Reactionless Mode Loads

In a 4-bladed rotor reactionless modes are excited by the second harmonic. According to the data given by Reichert the resonance factor of the second blade flap-bending mode is 2 for 2 P excitation. This excitation is usually quite strong and should lead to appreciable reactionless mode loads. It would be interesting to know whether the loads in the reactionless or differential coning mode have been observed.

#### 4 P Vibrations

In a 4-bladed rotor a 4 P vibration is transmitted to the body by 3 P and 5 P oscillations in the rotating frame. The data presented by Reichert show resonance factors of 3 and 8 for 3 P and 5 P excitation of the second and third flap-bending mode respectively. Neither of these load amplifications are obtained in the straight blade model. A realistic estimate of the 4 P vibration level is obviously not possible with this model.

#### Autorotational Loads

Usually the higher harmonic elastic blade loads are excited more in autorotation than in helicopter flight conditions. The straight blade analysis should be less accurate in autorotation. It would be interesting to know whether good correlations between analytical results and flight measurements have also been found for autorotation.

#### Pitch Link Loads

The agreement between the calculated and measured pitch link loads shown by Reichert even for an extreme flight condition is gratifying. This agreement would indicate that little coupling exists between blade torsion and the neglected higher flap-bending modes. In case of such coupling which could be used to reduce vibrations, one would expect less agreement between analytical and test results. In summary one can say that the straight blade model developed by MBB is basically a flight dynamics model for hingeless rotor-craft but can nevertheless be used to handle some aspects of dynamic loads, particularly for constant thickness hingeless blades. For other aspects a better structural representation of the rotor is required.

For the purpose of discussing the Lockheed loads prediction program I would like to make the distinction between a global dynamics model of rotorcraft and special purpose working models. The analytical model briefly described in the paper by Carlson and Kerraims to be of the global type, though it is presently still short of this goal mainly in the aerodynamic representation. Though such a global model has many useful applications it would be wrong in my opinion to consider it to be the principal tool for dynamic rotorcraft design and for dynamic load prediction. I believe, a careful balance in the applications of such a global model and of various special purpose analytical models should be worked out, and my comments will be mainly directed toward this problem.

#### Program Size and Execution Time Constraints

These constraints are mentioned though not detailed in the paper. Each flight condiction and maneuver requires a large block of computer time, typically 20 or more CPU minutes. Any design or trouble shooting effort will generally require variations of numerous parameters, which should be studied over the entire flight envelope. Even if the hardware is given and no parameter changes contemplated, it is good practice to study the effects of essential dynamic parameters in order to find out whether uncertainties as to their actual values, deterioration with operational time, or inaccuracies in the analytical model could result in overloads or in dynamic instabilities. The global model described in the paper by Carlson and Kerr is clearly not well suited for such purposes. Since the model includes a very large number of parameters, it usually is not very difficult, after a critical phenomenon has been observed in wind-tunnel or flight testing, to obtain reasonable agreement between the model output data and the observations by relatively minor adjustments. However, the goal of a dynamic design is to find a parameter combination with a healthy margin with respect to critical phenomena, so that the hardware will be free of critical loads or vibrations in spite of uncertainties or flaws in the prediction method. Because of the program size and execution time constraints it will be difficult to achieve this goal with a large global analytical model.

#### The Visibility Problem

The global analytical model described by Carlson and Kerr was obtained with the help of the Lagrange energy method and includes several coordinate transformations, from the feathered blade position to a reference blade position, from rotating coordinates to non-rotating coordinates. Presumably body fixed coordinates were used as is conventional in flight dynamics, which introduces many additional inertial terms for each mass element. Thus it will be hard to associate physical meaning with most of the terms occurring in the final equations or to separate the more important terms from the less important terms. Visibility of the major parameter effects is very important for a successful dynamic design, so that a number of less accurate but simpler special purpose models are likely to be more useful design tools. The main purpose of the global model, as I see it, should models.

## Balanced Sophistication

Great sophistication was used in the kinematic and structural representation of the rotorcraft, but much less sophistication in the aerodynamic representation. Large angle non-linearities and couplings from elastic blade deflections are carefully considered. However, the wake structure is rather crude, using inflow measurements for steady conditions also for unsteady conditions by merely adding a first order lag. The important and complex interaction between the tip vortex from one blade and the subsequent blade is not considered, unsteady aerodynamics with moment and lift hysteresis is apparently also not fully incorporated. It is true that many of these aerodynamic fundamentals are presently not very well known. This, however, raises the question whether a high degree of structural sophistication can be justified unless an equal degree of sophistication is available for the aerodynamic inputs.

Let me now add a few comments on the special problems mentioned in the paper by Carlson and Kerr which have been treated with the Rexor program. Once such a global program is available it is tempting to exercise it for many of the problems needing a solution.

Nevertheless it seems a legitimate question to ask whether or not at least some of the problems could not have been solved by a much smaller special purpose program.

#### Canopy Clearance Analysis

The main purpose of an analysis with full coupling between all blades is to include the transient modes and establish whether or not one of these modes can become unstable under certain conditions. As mentioned before, a greatly simplified analysis assuming that all blades perform the same motion with appropriate phase shifts, was found to be quite satisfactory for establishing trim conditions. The question I have is whether or not the problem of canopy clearance could not have been solved with adequate accuracy by such a greatly simplified analytical model.

#### Reactionless Mode Instability

Another application of the global model in the paper by Carlson and Kerr is to the problem of reactionless mode instability. Again the question comes to my mind whether or not a good approximation for this problem could not be obtained by a very much simpler special purpose model, possibly by a single blade analysis. It is known from single blade analyses published 15 years ago that for a chordwise stiff blade a structural or kinematic coupling which increases pitch with lag, the flap-lag motion can become unstable; while for a chordwise soft blade instability can occur when pitch decreases with lag. For a chordwise stiff blade upconing results in the unfavorable pitch-lag coupling, so that a flight condition with the highest coning angle will have the least margin with respect to flap-lag instability. Carlson and Kerr show that minimum damping of the reactionless mode was observed at about 20 knots which probably is the speed for maximum flapping because of maximum downloads on wing and tail, that the damping decreased with increasing gross weight and increased with increasing blade droop. All of this indicates at least qualitative agreement with a single blade analysis with its most important parameter, the pitch-lag coupling ratio. It would be interesting to compare the results of a single blade analysis with those of the global analysis shown in the paper.

#### Improvements of Global Model

In the conclusions the authors list a number of improvements which should be incorporated, like the aerodynamic effect of the wing and the body on the rotor inflow and the hysteretic blade stall effects. Both these effects can indeed cause large incremental blade loads. Their inclusion will make the global analytical model still more unwieldy, and the development of simplified special purpose models serving the dynamic design process becomes more urgent. Historically the global models did not have a very good prediction record, though after hardware experience with a critical phenomen they proved to be good at producing this phenomenon. Nevertheless, the global models will remain an extremely useful tool to spotcheck the range of validity of the special purpose working models.

PREPARED COMMENTS ON THE PAPERS

BY G. REICHERT AND R. M. CARLSON

AND A. W. KERR

Ъу

J. L. Jenkins NASA Langley Research Center Hampton, Virginia, U.S.A.

It is a great pleasure for me to participate in this Specialists Meeting on Helicopter Rotor Loads Prediction Methods, particularly, in view of the two excellent papers which were designated for my review. Knowing also that I followed Dr. Hohenemser, who unquestionably would provide the indepth comments on the theory of rotor structural analyses, allowed some latitude in framing my comments. I, therefore, have elected to present some general thoughts regarding these interdisciplinary computer programs as described by Mr. Kerr and Mr. Reichert, as well as to discuss some specifics.

Having been, on occasion, a user of some of these large computer programs which analyze the total air vehicle system, I am keenly aware of the inherent difficulties associated in their implementation. Thus, I was particularly struck by the analysis philosophy outlined by Mr. Kerr in his paper relating to the approach taken in developing the computer program. I fully endorse the specialist team approach as described by Mr. Kerr and the emphasis placed on the derivation of governing equations, the program structure, check procedures, data management, and documentation. Further, I would recommend Mr. Kerr's previous paper (Ref. 1 of his preprint) as being highly desirable reading, and a logical roadmap for anyone commencing the task of developing a comparable computer analysis.

The pitfalls of such a program are numerous, both for the initiator of the program and the user, especially when they are not one and the same. Consequently, the importance of adequate documentation cannot be overemphasized. This should include a clear delineation of the assumptions and limitations of the various subroutines in order to avoid the frustrating experience of wondering why and how certain portions of the computer printout ever came out like they did.

As we look at the complex structural system we are attempting to model analytically, as illustrated in Figure 1, and consider the even more complex aerodynamic environment from whence come the forcing functions, it is clear that "our reach can exceed our grasp," which is as it should be, with reservations!

With the introduction of large capacity/high-speed digital computers, we have tremendously increased the scope of the problems we can analyze. We have progressed from a simple, single, rigid-blade dynamic analysis to a single elastic blade, then to multiple elastic blade analyses, and now are progressing into flexible control systems and air vehicle dynamics. Along with this dynamics complexity, the aerodynamic routines have become perhaps even more complex. For example, stall, compressibility, unsteady aerodynamics, and wake effects all have entered into the analytical effort. With these sophisticated analytical representations of the total air vehicle system, care must be taken to insure access to the various subroutines through output techniques; otherwise, the user loses insight into the problem and can have as much difficulty establishing "cause and effect" relationships with a computer printout as with the output of flight test data.

One of the real technical advantages of a computer analysis is the ability to dissect a large problem into smaller subproblems which can be analyzed in their own right. The ability to exploit this advantage must be maintained in both the setup and output routines of the computer program. Just as there is difficulty in establishing the contribution of the many subproblems such as unsteady aerodynamics, vortex interactions, and aeroelastic effects in the flight environment, there is an equally difficult task in establishing the contribution of the subroutine solutions to the total problem unless adequate information and diagnostic routines are maintained.

We are all aware of the many technical disciplines wherein there is less than total confidence in our ability to analytically model them. Figure 2 illustrates just a few. It is, therefore, a difficult task to establish the "rightness or wrongness" of the many interrelated pieces of the whole. With uncertainties in the blade aerodynamics, wake effects, and structural routines there is the inherent problem of deciding wherein lie the errors. Thus, one of the major challenges for those of us engaged in experimental research is to devise unique and/or inventive tests which can provide the data necessary for verification of the varied technical disciplines applied in these air vehicle system analyses.

Another advantage of the total air vehicle system analysis is the ability to synthesize a flight vehicle beforehand, either to verify safety of flight or for concept evaluation prior to committing to a flight hardware program. The problem then arises as to what to use for input information and how sensitive are output results to discrepancies in these input parameters. Perhaps the two authors would comment on the problems of this type they have encountered when attempting data correlations and, particularly, on the sensitivity of output values to computer input parameter discrepancies.

As Mr. Reichert has pointed out, these interdisciplinary computer programs provide a large part of the information necessary for basic structural design loads. However, we are into an era of potentially significant civil application of rotorcraft, wherein a more stringent design criterion must be met. Here, I am speaking of the rather encompassing term, ride quality, where vibrations play an important part. Obviously, we must be able to analyze loads problems in much greater depth in order to bring forward with confidence a new generation of rotorcraft for civil application. Of course, we also have some ways to go in our ability to predict maneuvering performance and limits which, more often than not, are vibration limits rather than power limits. In this context, it would appear that improved vortex and wake theories

may well be essential. To illustrate, I have drawn from work done by Mr. Ward of NASA Langley Research Center and reported in the Journal of the American Helicopter Society in January 1971. His examination and analysis of rotor blade differential pressures and structural loads data point up some of the interrelated factors which impose a limit on the maneuvering flight capability of rotorcraft.

In the case considered, the data were obtained on a four-bladed rotor in limiting maneuvers. Illustrated in Figure 3 are planform views of the undistorted helical path of the four tip vortices laid down by the rotor in forward flight. The paths indicated three potential vortex crossings for a typical blade element, 0.95R in this particular case. The blade station was instrumented in order to ascertain the pitching-moment coefficients shown in Figure 4.

Data for two flight conditions are shown. The level flight (lg) pitching-moment coefficient, shown by the cashed line, indicates very little activity throughout the azimuth sweep of the blade. On the other hand, the data for the 1.5g pullup maneuver indicate very dramatic section pitching-moment excursions at the points where the vortex crossings are indicated. These three points are illustrated by the circular arrows. In level flight, the wake is apparently transported far enough below the rotor so that there are no adverse effects. In maneuvers, however, the curved flightpath alters this relationship and significant interactions occur.

There are, of course, other flight conditions wherein these interactions may be expected. In this regard, when the design criteria relate to vibration and maneuver limits, it would appear that wake models may be essential to the design analysis.

Mr. Reichert's paper has already referred to the free wake analysis by Mr. Sadler, published in NASA CR-1911. The program computes wake geometries, wake flows, and wake-induced velocity influence coefficients for use in blade loads calculations. A typical computed wake profile as tracked by the tip vortices is shown in Figure 5.

For the case illustrated, an investigation was being made to define the influence of nonuniform blade azimuthal spacings on vortex interactions and blade/vortex encounters. This wake analysis program was limited to level flight conditions and, as Mr. Reichert indicated, critical loads are determined by conditions such as maneuvers and gust penetration. Under a recently completed NASA contract, Mr. Sadler's work was extended to handle the wake flow in steady maneuvers and is reported in NASA CR-2110.

While the maneuver wake program has only been exercised in a somewhat cursory manner for checkout and demonstration purposes, it does predict tip vortex/blade interactions in the azimuthal regions and with frequencies as were expected for helicopters in steady maneuvering flight. It is hoped that more extensive utilization of this program will provide better insight into the complex interrelationship between the wake geometry, blade/vortex interactions, and dynamic stall. The ultimate objective being to confidently design to vibration criteria.

Allow me to change subjects now to some thoughts on the selection of hub precone for hingeless rotors. Mr. Reichert's remarks, in his text, regarding the normal procedure of establishing the design precone angle such that centrifugal moments offset the design thrust moments brought to mind some test data acquired on a hingeless rotor compound helicopter which illustrate the important roll precone can play in controlling loads. In the case of a compound helicopter, the design thrust moment is not as distinctly defined as it is for a pure helicopter. With the addition of a wing, the main rotor thrust is highly dependent on the flight velocity. Data typical of this trend are shown in Figure 6.

By virtue of the fixed collective-pitch mode of operation, the rotor gradually unloads with increasing flight speed, thus creating a completely different environment as far as the hub moments are concerned. To illustrate, the data shown in Figure 7 depict the hub moments corresponding to the thrust loading data of Figure 6. The maximum hub moments (i.e., the sum of the mean and the half amplitude cyclic moments) are plotted as a function of airspeed. Several load factor conditions are also shown. For these conditions, the centrifugal restoring moments obviously are the dominant loads for flapwise bending. Thus, maneuvering flight is accomplished at the higher speeds with less critical flapwise loads than in the level flight condition. For example, the flapwise loads in a 1.6g maneuver at 150 knots are no greater than those encountered at 110 knots in level flight.

Recognizing that the critical stresses are defined by combined loads, the designer has latitude in controlling the component loads. To do so, however, his design analysis must be valid and, of course, some forethought must be given to the mission profile for which the aircraft is intended.

In summary, let me again commend the two authors on their fine papers and, for that matter, all of the gentlemen who made this a most interesting and productive meeting. As exemplified by these papers, we are beginning to see significant improvements in our capabilities to mathematically describe and analyze the aerodynamic and dynamic complexities of a rotor. Unfortunately, as is so often the case, there are still some hurdles to clear. Rotor loads associated with deep stall, as encountered in severe maneuvering flight and vibratory loadings, have not yielded so readily to our analyses.

It has been said that the rotor is an excellent integrating device and to a large extent this is true when we restrict ourselves to time averaged events or events "in-the-large." As evidenced by the papers of this meeting, our problems are time dependent, unsteady, and dynamically complex. This puts a burden on all technical disciplines; the theoretician must retain the significant parameters in his governing equations which requires understanding and insight; the computer program must be viable both economically and technically and, most importantly, the experimentalist must devise the techniques to obtain data appropriate to the task of substantiating or verifying the analyses or at least, as a last resort, to provide the hindsight as to what should have been retained in the governing equations.

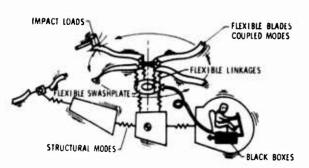


Figure 1. Dynamics complexity.

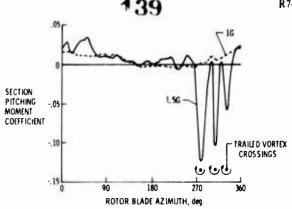


Figure 4. Aerodynamic pitching moment.

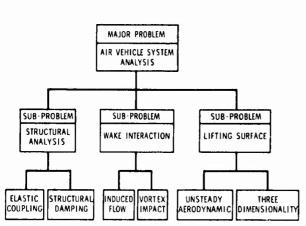


Figure 2.

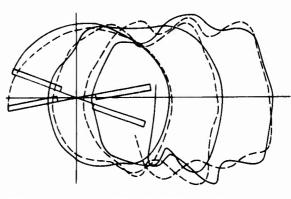


Figure 5. Wake geometry prediction computer program.

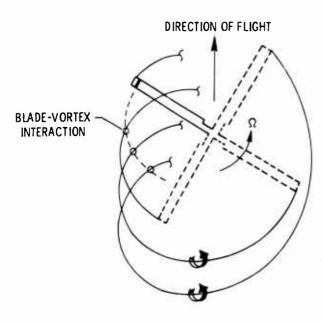


Figure 3. Trailed vortex pattern.

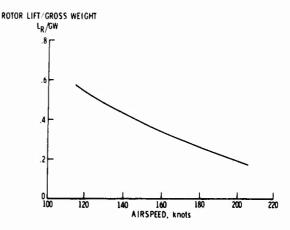


Figure 6. Level flight rotor lift variation.

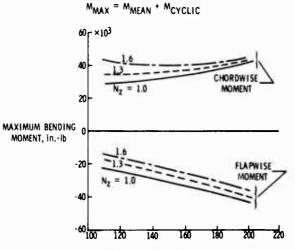


Figure 7. Maximum hub bending moment.

# Discussion of Paper 7 "Load Prediction Methods for Hingeless Rotor Helicopters" presented by G.Reichert

A.Z.Lemnios: In the written version of your paper, I noticed you have a figure 12 that you did not show on the screen. This one shows the pitch link loads of the BO 105 with 0012 airfoil profile and a 23 012 airfoil profile.

Which of those two airfoils are now being used on the BO 105?

G. Reichert: The cambered airfoil.

A.Z. Lemnios: Can you explain why the 23 012 gives lower pitch link loads than the 0012?

G. Reichert: For the higher thrust condition the difference is mainly affected by the shift of stall to higher loads. The overall difference in conditions without stall effects is due to trailing edge modifications at both profiles and a slightly modified blade e.g. In addition, especially for a hingeless rotor, small changes in the blade loading situation will influence pitch link loads. A small change in the loading situation will result for example from a slightly different lift slope of the airfoils.

W.Z.Stepniewsky: I would like to ask you one question. How did you obtain your experimental maneover loads? Whether in basically transient symmetrical pull up or in steady state turns.

G.Reichert: The data I showed here are based on turns but we have also data of pull-up maneuvers.

We did flight testing and calculation work also for maneuvers such as the UTTAS requirement as mentioned in the discussion yesterday. Also for this case the correlation was good.

## Discussion of Paper 8 "Integrated Rotor/Body Loads Prediction" presented by A.W.Kerr

**P.J. Arcidiacono**: Could you tell me what kind of running times are associated with your programme for different applications?

A.W.Kerr: The application with which we have the most experience right now, as you might imagine from our flight experience, is in assessing reactionless mode stability.

This requires establishing a trim condition and approximately 8 seconds of response time. Currently, a run like this with an 8 second dynamic maneuver response requires a little over 20 minutes to run.

This is very close to the same sort of time that was experienced on the C-81 program. I would say that times are quite comparable between the two programs.

## REVIEW 6 by K.Hohenemser Reply by G.Reichert

I only want to give a few comments. The first concerns the representation of the elastically bending blade by an equivalent hinged straight blade. I have here a diagram showing the first flapping mode, and two straight blade lines for different hinge locations. We can match the equivalent systems to be very close to the bending line at the outer part of the blade, and this part of the blade is the important one concerning the aerodynamics. The two systems shown are both dynamically equivalent, they have the same frequency (by hinge offset and hinge restraint) and we can make sure that we can get a good aerodynamical representation. With proper selection of the equivalent hinge offset and hinge restraint there will be only a very small difference if you will consider a modal representation or the straight line representation. On the other side it would not be too difficult to include the real mode in the analysis but we think it would not be necessary in this special case.

I would like to comment to the generalized forces and the amplification factors. Maybe that there is not complete information in the diagram shown in the paper. The model which is used is only a single degree of freedom system presenting the blade mode by an equivalent mass, an equivalent spring and an equivalent damper. In addition

to the generalized forces we have to consider the equivalent springs for a final comparison of the resulting moment. These are, of course, different for the different modes, and this perhaps would make it more clear.

Concerning the question of the representation for autorotational conditions I have to mention that we have a lot of autorotational flight test data. It is true that the correlation is not as good as in normal flight. These autorotations show some more higher harmonic loads. A similar situation occurs for low speed flying conditions.

To investigate reactionless modes or second harmonic modes we did some flight testing with instrumented blades with 90 degrees shift of azimuthal position. There is some difference, but second harmonic loads are not very high.

Concerning the question about the pitch link loads, I agree that the coupling effects due to torsional and blade bending effects are not very strong for the normal BO 105 blade.

#### Reply by A.W.Kerr

Just a couple of comments. At least now we can say that we do not have a torsion mode. We have added a full dynamic torsion mode since the paper has been written. One of the reasons that it was not included originally was that we did include the elasticity in the control system and the pitch horns as degrees of freedom, both of which are the major contributors to low fre juency dynamic blade feathering motion. The torsionally stiff rotor blades themselves have a much higher frequency, but we have found it desirable to include as an option a full representation of the blade torsion itself.

I have to agree with Dr Hohenemser that there is a definite visibility problem which comes from using the Lagrangian approach. One way that we can get insight into what is going on is by comparing perturbation matrices with the linear equations that we derive from linear analysis where equations of motion are derived from a Newtonian or d'Alembert approach. By doing this we have found errors in both types of analysis. We have found terms that have been left out of our strictly Newtonian approach when we have used the complete analysis to generate our full matrices. One of the problems that we have with the large analysis in its earlier version was finding missing terms from time to time, so we have elected to go to the Lagrangian method in the derivation of this model.

In terms of canopy clearance, I also agree with Dr Hohenemser that the problem could be studied with a single blade analysis or one with fewer modes. As Dr Hohenemser has pointed out, the reactionless mode and its coupling with the gyro control system requires the complete analysis to estimate the damping which is present in this system. We have not succeeded in getting good correlation using our standing linear analyses, and although we feel that eventually we will be able to develop a linear model that gives better correlation than we have right now. We now rely on our extensive mode damping for our rotor system.

## REVIEW 7 by J.L.Jenkins Reply by A.W.Kerr

I would like to speak of the one question which Mr Jenkins asked about the large computer models, and that is what our experience has been with the sensitivity to input parameters when we are trying to do correlation. The problem of correlation of total vehicle models is to get all elements to correlate simultaneously. We all know that we can bring in a correlation that will look good in a paper, but it has been my experience that there are so many variables which must be either preset or adjusted in order to make sure that, for instance, the flight conditions on the rotor are precisely the way they are on the test aircraft. As a result, it is quite often easy to get correlation for particular parameters. However, if you examine the total vehicle, and look at all of the data you can get for a particular flight condition, you find that having forced the solution so that the rotor blade loads are correct, something is incorrect on the body or vice versa. So, it is true that there is a significant problem in correlation, and that problem is compounded when you start to try and correlate various extreme operating conditions and maneuvers where everything is changing including the loading conditions on the rotor. This is an area which is of great concern to me in terms of establishing a correlation criteria and determining when a correlation is adequate.

The one other point that I wish to mention is that I think that the vibration problem in terms of pilot and passenger comfort as it affects the commercial aircraft is going to be solved because of the new requirements which the Government is placing on military aircraft in order to provide the maximum .05g vibration level that was mentioned earlier. This is a fairly new requirement, and it is an area where I think that industry is beginning to expend considerable effort. I think that there is going to be a great deal of progress in this area in the near future in order to meet these requirements.

#### Reply by G.Reichert

I fully agree with Mr Kerr and his comment about the sensitivity of the input parameters and I feel that this sensitivity is stronger for hingeless rotor helicopters than for a helicopter with an articulated rotor because the hingeless rotor is much more sensitive to small control inputs or to small differences of parameters.

I would like to give a comment concerning the precone angle selection. Normally, the precone angle will be chosen for unloading with the design rotor thrust, but for a hingeless rotor the blade inplane stability situation may be more important, because the precone angle is strongly influencing inplane damping characteristics in a sense that a low angle would improve stability and a high angle could lead to an unstable situation. The situation is not as critical as sometimes is believed. For the BO 105 the precone angle is 2½ degrees. This value is about that of design thrust. There are no inplane stability problems all over the flight envelope.

#### SUMMARY ANALYSIS

Robert G. Loewy
College of Engineering and Applied Science
University of Rochester
Rochester, New York 14627

At the beginning of this meeting our chairman, Professor Thielemann, in his opening remarks charged the summarizer to consider what progress has been made recently in rotor loads prediction methods; what the implications of such progress are for helicopter capabilities, reliability and cost; and finally, what remains to be done. I will attempt to do that.

What has been accomplished to date? From what we have heard, it would seem that our ability to predict helicopter rotor loads has progressed. In keeping with Professor Stepniewski's admonition not to get the mathematics too far ahead of the physics, I'll say, first, that we've increased our understanding of the physics that is involved. In addition, we have also increased our facility in applying the mathematical methods that must be used. And we have began to reduce the cost of conventional rotor designs by these prediction methods and to decrease the risk in the design of unconventional rotors by using our increased analysis capabilities.

Let me take some of these things step-by-step. First the physics. The physics involved in rotor loads, as most of our speakers have said, can be considered in two areas, structural dynamics on the one hand and aerodynamics on the other.

The structural dynamics of rotating flexible bodies—as one of our reviewers, Ray Piziali, indicated—involves nothing very new. One might say, "nothing new since Isaac Newton." However, Dr. Gaymann emphasized helping the designer, the manufacturer and the man in the maintenance shop. It is not possible to approach complex structural dynamics problems with that attitude and hope to proceed like a researcher. The researcher knows precisely the operating condition to be analyzed and usually has the results to be matched. He usually has enough time to attempt correlation, and when something doesn't "match," to go back again and find the thing that has been left out. He often has the time to begin with a clean piece of paper and say, "I'm going to start from the beginning and include 'everything' in my analysis." If we are to help the designer, manufacturer, and maintenance man involved in projects where correlation isn't the game, but "making it work is..." with a set, usually too-short time scale, and with a set, usually too-small budget...then the approach must be different. We usually can't begin with a clean piece of paper, but must draw on experience. This generally means limiting the number of degrees of freedom, leaving out some effects that we feel will not be important, and being satisfied with the accuracy of the characterization of springs, masses and dampers as they are known at the time. Clearly, one has to be very careful that all the important structural dynamics aspects are in fact included. Just how difficult this really is, as I said from the floor earlier when structural dynamics were being discussed, is evidenced by the fact that, where aircraft have gotten in serious trouble, where things have fallen apart—literally fallen apart....it has been structural dynamics aspects that have generally been at the root of the problem.

What have we learned in the structural dynamics area? I think we are now alert to some things that were often obscure in the past. I'll try to recite a few. First, twist couples flat-wise and edge-wise bending. Second, large initial flat-wise bending deflections combined with Coriolis forces usually produce large torsion moments. Third, large initial edge-bending or "sweep-back" is kinematically equivalent to 3, the coupling between flapping and pitch. The list may be much longer, but here's just one more; the presence of large twist and 3 produces the equivalent of kinematic coupling between lag and pitch. These are some of the things we are more alert to now than we were five or ten years ago, and in certain cases any one of them can be crucial. We heard about another phenomenon today from Mr. Bennett which I hadn't heard previously and that is "cyclic detuning"; this aeroelastic effect may also prove to be quite significant in some cases.

Let's turn to the aerodynamics area. What has transpired aerodynamically in the last decade of rotor developments? First, ten years ago it was uncommon for an azimuthally non-uniform inflow to be accounted for, i.e. by integrating wake-induced effects. Now this is common. We are also beginning to discard more routinely the assumption that the vortex wake is rigid. Thus, we have made progress as regards the induced velocity fields in which a rotor blade operates. We have also become more aware of the intricacies of vortex-airfoil interactions, and are now, at least, sophisticated enough to know that we should be worrying about whether we can get away with assuming a lifting line or must use lifting surface theory—whether we can use discrete shed vortices or whether we must use a continuously shed vortex sheet. In the data we use for our airfoil section characteristics, we have more insight as to how fully stall must be represented to evaluate accurately things like pitch link loads, and we are also sophisticated enough to know that stall evidences itself sometimes separately—i.e. under different conditions—in lift, drag and pitching moments. We know that the aerodynamic effects of sweep and span-wise flow are important. In all these ways, we have advanced.

Turning from these physical matters to the mathematical area, it is clear that the use of computers has allowed us to take "a great leap forward"—to coin a phrase. We use digital computers to calculate natural frequencies and modes very routinely, not only for uncoupled modes (that's been going on for many, many years) but now for "fully-coupled cases," by which we generally mean coupled flap bending, lag bending and torsion. This is now becoming routine. Although some people have been doing it for some time, the practice of including the effects of flexibility and inertia in the stationary system on the natural frequencies, modes and loadings of structures in the rotating system is becoming more widespread. The power of the computer has

also allowed us to make routine use of "table look-up" routines for aerodynamics....mach number, unsteady, and large angle effects, etc.....It is now possible and often practical to do step-by-step integrations in a way which was out of reach before the advent of large-scale digital computers; non-linear effects are now, therefore, routinely handled by such step-by-step integration around the azimuth. The computer has allowed us to "close the loop" with iterative schemes which were not practical without them. We use iteration now at virtually all levels of sophistication. We use it, for example, to solve the non-linear aerodynamics problem, then calculate the aero-elastic response, and then go back to the aerodynamics portion, correct for motions and iterate until, hopefully, convergence. Iteration is also used routinely to satisfy boundary conditions for components....such as at blade roots....and for the entire aircraft, as occurs in "trim" calculations. Where at one time we would have had to guess at control inputs and be satisfied with the moments and forces that resulted, now we go back and forth between them until the proper balance is achieved. Finally, we have begun to "close the loop" with iteration in design problems, as we heard from Andy Lemnios; i.e. design parameters are being varied and the performance calculated iteratively until the design has been optimized.

The mathematical difficulty of dealing with non-uniform, non-constant coefficients in our differential equations is also giving ground. The use of Floquet theory to identify the special kinds of instabilities encountered in forward flight as a result of the varying aerodynamic effects as the rotor sweeps through the azimuth is becoming routine. And we have just begun, because of this increased computer power, to admit that perhaps one blade on a given rotor does not behave exactly like another on the same rotor. We are now able, despite the increased number of degrees of freedom that this implies, to put as many blades separately on the hub as may be necessary.

We are all agreed that the use of large-scale digital computers is hardly an unmixed blessing, and some arguments for limiting our reliance on them were eloquently expressed by Professor Hohenemser in terms of working models vs. global models. Experience suggests that no model is so global that you can't leave out the most important effect. On the other hand, the position taken by Mr. Kerr is equally supportable; the level of judgment needed to include the important effects in a working model is an order of magnitude greater. Thus, both global and the working model approaches should go forward together--and in fact they usually do.

A similar argument arises between LaGrangian and Newtonian approaches. It's no good to simply "take what you get" from a LaGrangian approach and use it blindly. On the other hand, until experience is so complete that you can always be sure that "you've got it all," it would be a mistake not to use the LaGrangian approach as a means of checking the results of a Newtonian approach arrived at using intuition and/or experience.

As another part of the mathematical approach, there is the question as to when a particular calculation should be a "pre-calculation" and when it should be an inherent part of the overall rotor load prediction program. No universal agreement exists in all cases, but there are some where there is a general feeling of security in the approach chosen.

First consider the mathematical idealization of the hardware to be analyzed. In Andy Lemnios' paper, he spoke of the "reconstitution step." This has become, in some instances, part of the overall rotor load prediction method. In others it is a pre-calculation. In one approach reported by Dick Gabel, discrete masses, connected by beam elements, are individually used as degrees of freedom whereas, in others, a select group of calculated natural modes and frequencies are used as generalized coordinates. Making use of the natural mode pre-calculation, requires a subsequent choice among the resulting modes and frequencies for generalized coordinates, and this presumes considerable experience; leaving out one of importance is a common source of serious error. This argues for the discrete mass as opposed to generalized coordinate model in unfamiliar cases.

Airfoil data is generally assumed to be something that results from a pre-calculation or from a test, not something calculated as part of the general prediction method. That it should remain so is a point made very strongly by Mr. McCroskey and is fairly well agreed to within the rotary-wing community.

Wake geometry and induced inflow are generally determined in pre-calculations in rotor load prediction methods, but in distorted wake analyses, iteration is bringing these matters increasingly into the overall procedure.

Thus, it would seem that in both physics and mathematics there has been progress. But what has this done to improve the state of the art in rotor design? We are able to predict bending moments at higher harmonics and at lower forward speeds better than we did before. We are beginning to predict control loads accurately, even into stall. I think this is illustrative of real progress. We saw some data even in this meeting in which the best efforts to match control loads weren't very good, but we also saw some where correlation was good well into stall. We are able to predict vibratory hub forces and moments with the beginnings of some certainty. The very stringent vibratory-"g" requirements that John Shipley spoke of will only be met if we have the ability to predict vibratory hub loads early enough in the design cycle to allow taking the steps necessary to reduce those loads.

We have begun to use the results of rotor load prediction methods, as I alluded to earlier, in design directly; that is, we have begun to optimize rotor designs based on a balance among performance, power requirements, oscillatory hub loads for vibration reasons, and oscillatory loads on the blades themselves and in the control links, from stress considerations. In one case --Mr. Bennett's paper--we heard that a very complex analysis such as C-81 is being used on an "open-shop" basis by designers to determine control requirements. Furthermore, although I have not conducted such a study (and I don't know that one has ever been made), I have the impression

that progress in these methods has in fact reduced the cost of developing new rotors. Instead of running into unexpectedly high loads almost everywhere the first time the full flight envelope is explored, we now only run into them occasionally, at some extreme flight condition.

What still needs to be done? Again, let me speak of the physics, the mathematics, and design. In the aerodynamics area the vortex wake still is a problem for us. Here is a case where the physics is not being looked at carefully enough in my opinion. We heard from Mons. Gallot that perhaps the stability of the helical vortex trail beneath rotors ought to be more carefully considered. I agree. As a matter of fact there are recent publications by Sheila Widhall at Massachusetts Institute of Technology, and by Gupta and someone named Loewy at the University of Rochester, that shows that not only are there one or two unstable modes for a helical vortex, but in fact there are only one or two stable modes. That is, a continuum of modes exists in which the vortex will distort, tending to obliterate the initial helical pattern. We need still more airfoil data than is now available for unsteady stall characteristics, and for unsteady compressibility effects. Certainly the transonic characteristics Mons. Hirsh spoke of should be examined more extensively.

In terms of airfoil-wake interactions, we  $\underline{do}$  have to decide when to use surface theory and when not to use it. We need to know more about the size of the vortex core. Its dimensions will influence the kind of fluctuating angle of attack variations and induced resultant velocity variations experienced by an airfoil in passing close to a vortex. Finally, as regards aerodynamics, the question has been raised as to the aerodynamic differences between  $\underline{\bullet}$  and  $\underline{\lambda}$  effects.

Turning to structural dynamics, the case has been made very well by Professor Hohenemser and it has been implicit in virtually everything that has been said here, that the method must be well suited to the case at hand. In the Lockheed paper a torsion representation was used which contained no inertial effects. While that was later revised, the initial assumption worked out reasonably well. That could certainly not have been satisfactory for the Kaman rotor, which had a natural frequency in torsion of 3 C (three per rev.). As another example, one flexible flapbending mode was sufficient in the Kaman case, and no lag-bending mode needed to be represented. This certainly would not have been adequate for the cases presented by Vertol-Boeing, where the second and third flap-bending and the first and second lag-bending modes all proved to be important. Thus, matching the method to the particular situation is a matter of the utmost importance.

Swash-plate dynamics have been alluded to. These are generally not currently included in our analyses. We know that swash-plates and their supports are not only flexible, but are anisotropically flexible, and often add damping to the system. This in fact may require that, for some analyses, we will have to provide separate degrees of freedom for each blade. The criteria for this decision are not yet clear. Certainly the kind of blade-to-blade couplings that result from flexibility of the swash-plate gives us some pause. Other aircraft couplings have to be explored thoroughly enough to find out when we must include them; for example, when to include fuselage-hub degrees of freedom and when we needn't include them.

And finally, as regards what should be done in the "physics" category, we probably need more emphasis on diagnostics....a point made by Mr. Piziali. Simply asking if the end result compares well with the quantity we expected, is rarely good enough. We usually need to know some details; what goes on between input and output? This includes things like chordwise pressure distributions. It also includes attempts to correlate, not just the peak-to-peak stresses at a given location, but complex response, that is, phase and amplitude, of each mode to each of the various harmonics of the forcing functions.

What efforts are called for in the area of mathematics? I believe that we need to begin more of our iterative schemes with the solution of the equations which are obtained by linearizing the pertinent non-linear case. It is unnecessary to begin a forward integration from scratch. We usually know how to solve the linear problem, so why not apply those linear solutions to the first steps of the integrations? This is done frequently in the structural dynamics area. It is not done at all in determining the wake geometry, so far as I know. Our first step in solving problems involving the wake is usually to assume uniform downwash, constant tip speed and forward speed; a helical wake of a given pitch and inclination results. Actually, we need not be limited to that starting point. Both forward flight and hovering applications of momentum theory exist and will lead to a configuration for the trailing vortex which is far from a helix and much closer to reality. That can be used as the starting point.

Mr. Sorbey's suggestion that we should have a new look at non-linear solutions to complex non-linear problems is a point well taken. Perhaps we can handle these things better than by the brute-force approach of "forward integration." Certainly in the case of differential equations with non-constant coefficients we don't simply use a forward integration. We have begun rather commonly to use Floquet theory. This, by the way, might be profitably applied as a way of evaluating the "cyclic detuning" we heard about from Mr. Bennett.

Finally, Professor Stepniewski called attention to the question of when to use probabilistic methods in favor of deterministic methods. The scatter encountered in flight tests certainly suggests that some of the things we have considered deterministic may in fact not be so. We may find it profitable to consider more thoughtfully whether the process of interest calls for stochastic or statistical approaches and make a conscious decision as to the proper methodology to apply.

In the analyses intended to support design, we need to develop maneuver load and other transient load prediction methods more suitable for design purposes. At the moment these cases are handled as very special, painstaking calculations. We have not yet got our tools for handling them to the point where they can be used routinely and with confidence. In this same general area of design, it is also important that we begin to automate more, taking fuller advantage of

the computer to facilitate the application of our analyses to optimum design. We seem to have had a beginning at this and it should be pursued vigorously.

In concluding these remarks I offer the opinion that this meeting has brought to this group of experts a much more complete awareness of what is going on in other companies, other laboratories and other countries than they would have otherwise enjoyed. This is bound to be useful. I have learned from it, and I feel certain that if all of the participants here have not been made aware of something completely new to them, they have at least been stimulated to some new ideas of their own. It is not possible to state with confidence when it will be profitable to hold such a meeting again, but it seems to me that we should be thinking about it, and as soon as further advances have reached a point when it will be worthwhile, a similar meeting should be held again.